

An Alternative Base Case for Modeling Trade and the Environment

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Abstract

The trade-and-environment literature often begins with a generic base case. Sectors differ in pollution intensities and countries have sectoral comparative advantage. Pollution comes from production, not consumption, and consumers have identical, homothetic preferences over goods and environmental quality. Policy instruments are Pigouvian taxes and tariffs. I argue that this model is counter-empirical in some ways and assumes away other crucial features. I offer a simple alternative base case, in which global pollution is proportional to all economic activity, and where environmental quality has an income elasticity of demand exceeding one. An income tax can fund an abatement activity, diverting resource from production and creating a terms-of-trade externality that passes some costs of abatement to the other country. I contrast cooperative and non-cooperative outcomes between governments of two countries differing in per-capita incomes. Free trade is good for the environment because (a) gains from trade increase resources devoted to abatement and (b) countries can pass more of their abatement costs to the other country in the non-cooperative case. Leakage takes a very different form called “policy leakage” and border taxes are counter-productive. When per-capita income differences are large, a poor country may be worse off when the large country abates (reversing the usual argument on free riding) and cooperative bargaining over abatement levels may offer no gains. Finally, I examine “issue linking” in bargaining when one country is both large and rich, and hence has both a high tariff and a high abatement effort in a non-cooperative equilibrium.

I am primarily a trade economist and have been away from environment issues for a couple of decades. So I am currently seeking and reading related papers. I am especially interested in finding empirical work that estimates something like an income elasticity of demand for environmental quality. Please send references with attached notes explaining their relevance. Self-references are most welcome.

The first draft was finally written while a visitor at Hitotsubashi University in 2012, and I thank my colleagues there for a delightful and productive three months.

April 2015

1. Introduction

Much of the literature on trade policy concentrates on the production side of general equilibrium. In the trade/environment literature for example, a typical model might involve the nexus among factor intensities in production, factor endowments of countries, and pollution intensities across goods. A common question might be how changes in trade or environmental policy of one country then change total world pollution and the distribution of pollution emissions across countries (e.g., the carbon leakage and pollution-haven literatures). Instruments controlled by governments are generally Pigouvian corrective taxes and tariffs.

Considerations of the role and determinants of the demand for environmental quality across countries and hence endogenous policy are less featured, with the exception of a few papers on the environmental Kuznets curve. Per-capita income differences have, of course, played a prominent role in climate negotiations (most recently in Poland), with poor countries arguing again having to bear restrictions while high income countries want an even burden sharing. Many of the arguments strike me as playing at ethics, such as arguing whether or not the high-income countries' historic contribution to the stock of atmospheric CO₂ means that they should bear much of the burden. Economic analysis seems lacking.

A principal focus of the paper is on how the per-capita income levels of and differences between two countries affect their abatement efforts in a non-cooperative policy-setting game. This outcome can then be used as a disagreement point to analyze Nash bargaining to solve for a cooperative outcome. To attack this problem, I develop a model of non-homothetic preferences in which environmental quality has a high income elasticity of demand. A second focus is to move away from sectoral pollution intensities and Pigouvian taxes. Global, trans-boundary pollution depends only on both countries' total production and resources withdrawn from production are used in an abatement activity, financed by a non-distortionary tax. Comparative advantage in two consumption goods links the countries through trade, such that more abatement by either country shifts world commodity prices in that country's favor by essentially shrinking the country's economic size.

After developing the general two-country model, I present a simplified version which I call the "model of the model", with which we can derive analytical solutions for best responses by each country to the other country and how these best response functions and the non-cooperative Nash equilibrium relate to the levels of and differences between per-capita incomes in the two countries. Following this, I develop the a general simulation model using a relatively novel optimization solver in GAMS. I model the general-equilibrium policy problem as an MPEC (mathematical programming with equilibrium constraints). In a non-cooperative outcome, each country chooses an abatement tax, and a tariff in a later section, to maximizing its utility given the tariff and tax of the other country, with the two-country general-equilibrium model as a constraint set. Cooperative outcomes can be found using non-cooperation as a disagreement outcome, which I contrast to other reference solutions for world welfare.

A couple of basic results are apparent in the analytical model, illustrated again in the

general-equilibrium model. One result is that optimal abatement policy produces a non-monotonic Kuznet's curve. As per-capita income rises from a very low level, environmental quality falls, but later rises as an abatement policy is instituted, with the share of income devoted to abatement increasing in per-capita income. Second, non-cooperative policy is characterized by "policy leakage"; that is, a government will reduce its abatement effort in response to an increase by the other country. Third, an interesting result emerges when the two countries are identical and have a strong pattern of comparative advantage. Because an abating country passes on half the cost to the other country in a non-cooperative equilibrium, it turns out that the non-cooperative and cooperative outcomes are identical in a special case.

I then turn to simulations to analyze more complex cases of cooperative and non-cooperative outcomes (the model has 28 dimensions). Free trade is good for the environment because (a) gains from trade increase resources devoted to abatement and (b) countries can pass more of their abatement costs to the other country in the non-cooperative case. Allowing the per-capita income of the countries to differ holding total income constant (e.g., one country has fewer but more productive households), the Nash non-cooperative policy outcome has a higher abatement effort by the high-income country and a lower effort by the poor country relative to when both countries have identical per-capita incomes. The same is true for a cooperative Nash bargaining outcome using the non-cooperative outcome as the disagreement outcome.

Two further experiments are conducted on this question. In the first alternative to the base case, comparative advantage is eliminated so that there is no trade and no effect of one country's abatement on the terms of trade. This reduces the non-cooperative abatement effort of both countries, since a small increase in abatement no longer shifts the terms of trade in that country's favor. I note that it can now be the poor country that gains more at both the cooperative and non-cooperative outcome relative to zero abatement by both countries (it does not have a terms-of-trade loss).

In the second alternative case, the difference in the per-capita income of the two countries is widened considerably (returning to base-case comparative advantage). One interesting result is that the poor country may be made worse off by an optimal abatement effort by the rich country, since the term-of-trade deteriorating for the poor country may outweigh the smaller benefits of any spillover improvement to its environment. This case also illustrates an important policy point, which is that there may be no gains to cooperation if abatement efforts are the only instrument available to the two countries in their bargaining. Another result that relates to this is that there is no role for "border tax adjustments" in the model. If one country places an (exogenous) tariff on imports, that only makes its trading partner poorer, and the latter country reduces its abatement effort.

A natural extension of the analysis is to consider "issue linking" in international bargaining. Specifically, there is the standard motive for protection in that an import tariff improves a country's terms of trade. Is there a role for linking together bargaining on abatement efforts with trade liberalization? A clear possibility occurs when the rich country is also large, so that it will have a high tariff in a Nash equilibrium in tariff rates, as well as a high abatement

tax. I present a case in which linking helps the small poor country without harming the large rich country, but bargaining theory suggests that robust results are unlikely.

Before turning to a brief literature review, two motivating data plots are given in Figures 1a and 1b. Figure 1a plots an environmental sustainability index from the Global Competitiveness Report against the log of country per-capita income. While there is variation in the data, it is clear that richer countries have more sustainable policies despite having a lot more production and consumption. Figure 1b motivates my desire to move away from focusing on the relative pollution intensities of traded goods. In the US, for example, only about 34 percent of all energy is consumed by “industry” and this includes agriculture and non-traded manufacturing. The share of industry is higher for the world as a whole, but in a survey article Sato (2014) reports that literature estimates of the share of carbon emissions embodied in trade is somewhere in the 15-30% range with a large variance.. Further, Figure 1b casts doubt on the standard assumption that pollution/energy use arises from the production of good. Clearly, a lot arises from consumption, which makes a big difference in open-economy models.¹

There is an immense amount of literature related to this paper, in part because it overlaps a little with three quite separate agendas. First there is the trade and environment literature. Second, there is a literature on income elasticities of demand generally and willingness to pay for environmental goods specifically. Third, there is a literature on bargaining and linked games. I have done a little work in all three areas but am an expert in none individually. I’ll offer a short review, but will surely miss some things that are important. Please let me know.

The trade-and-environment literature is extensive, but few papers seem to include any theoretical consideration of per-capita income and non-homothetic preferences. Clearly, Copeland and Taylor’s (2004) review article is a starting point, with other features in Copeland and Taylor (1994) and (2005). Papers embedding non-homothetic preferences in general-equilibrium trade models include Hunter (1991), Hunter and Markusen (1988), Markusen (1986, 2013) and Caron, Fally and Markusen (2014). There is a significant empirical literature on the environmental Kuznet Curve, relating environmental quality (or inversely pollution) to per-capita income or some other measure of development. Papers include Chiu (2012), Deacon and Norman (2006), Grossman and Krueger (1995), Selden and Song (1994) and Tang (2012). Evidence for a non-monotonic Kuznets Curve (environmental quality at first falls, then rises with per-capita income) seems mixed, but there are many separate factors at work other than simply preferences.

¹By pollution from production versus consumption I mean, for example, energy used in building a car versus energy used in driving the car. The literature virtually always assumes pollution from production, but has a quite different production-versus-consumption issue: does the pollution from production get “charged” to the country where the car is made, or to the country where it is sold (Sato 2014). In my model, both latter issues are moot: at world prices, the pollution content of a country’s consumption equals that in its production, and similarly the pollution content of its import bundle equals that of its export bundle given trade balance.

While Pigouvian taxes (with revenues redistributed to consumers) dominates a lot of the analysis, a few papers model real-resource-using abatement activities, including Copeland and Taylor (2003) and Bogmans (2015). The terms-of-trade effect I identify here has a closely related parallel in Bogmans (2015), where an emissions standard has an “iceberg” effect that shrinks output (the abatement activity uses the good itself). Both Bogmans and the present paper imply that abatement shrinks a country’s output in a neutral way and hence generates a positive terms-of-trade change in the country’s favor.

On the environmental economics side, there is a literature examining the income elasticity of demand for environmental quality and the income elasticity of willingness to pay. These are not the same as noted by Flores and Carson (1997) and it will be true in my model that the income elasticity of demand exceeds the income elasticity of willingness to pay (the latter equals one). Evidence that these income elasticities are greater than one is mixed: evidence is presented in Hökby and Söderqvist (2003) and Kriström and Riera (1996) casts some doubt on this. Pearce (2006) summarizes evidence from a number of studies and, while there is a lot of variance, provides some support for environmental quality being an income-elastic good, a finding more strongly supported in Deacon and Norman (2006). Navrud and Strand (2013) focus on the income elasticity of willingness to pay, and find these elasticities less than one using “raw” GDP per capita but greater than one using PPP correct GDP per capita. Again, in my model the income elasticity of willingness to pay is equal to one and less than the income elasticity of demand for environmental quality.

Finally, there is a general literature on cooperative and non-cooperative policy outcomes and issue linking in trade policy generally (in turn drawing on pure-theory literature which I will not discuss) or in a few cases explicitly on environmental negotiations. Literature includes Abrego, Perroni, Whalley and Wigle (2001), Bagwell and Staiger (2001), Conconi and Perroni (2002), Gori and Lambertini (2013), Harstad (2012a, 2012b), Horstmann, Markusen and Robles (2005), Markusen (1975), Markusen, Morey and Olewiler (1995), and Markusen and Wigle (1989), and Spagnolo (2000). I find it hard to draw general conclusions here. It is certainly the case that issue linking can result in Pareto improvements for negotiating partners: there are more instruments to hit multiple targets (trade barriers, environmental protection). But it is also not hard to produce special cases where one country is worse off by linking.

2. Production, Income and the Environment

In this section, I consider an international policy question that is of current interest: the relationship between trade policy and international environmental policy with a global pollutant such as CO_2 . Assume that we have two final consumption goods (X_1, X_2), one environmental good (E), and two countries (n, s), where n (north) will be the higher per-capita income country.

Final goods are produced by a CET (constant elasticity of transformation) technology, with one input L , with L_x denoting the total efficiency units of labor allocation to production.

$$\left(\sum_i \alpha_i \left(\frac{X_i}{\alpha_i} \right)^\beta \right)^{\frac{1}{\beta}} = L_x \quad \infty \geq \beta \geq 1, \quad \sigma = \frac{1}{\beta - 1} \quad i = (1, 2) \quad (1)$$

$$\left(\sum_i \alpha_i \left(\frac{X_i}{\alpha_i} \right)^{\frac{\sigma+1}{\sigma}} \right)^{\frac{\sigma}{\sigma+1}} = L_x \quad \sum_i \alpha_i = 1$$

where σ is the elasticity of transformation along the production frontier. The alphas, normalized to sum to one, are parameters that indicate comparative advantage when they differ across countries. Use n and s superscripts to denote countries, $\alpha_1^n > \alpha_1^s$ (and therefore, $\alpha_2^n < \alpha_2^s$) indicates that the north has a comparative advantage in good X_1 and the south has a comparative advantage in good X_2 .

This transformation function is presented in an appendix to the paper, which also shows two further results. First, the unit ($L_x = 1$) revenue or national-product function is given by

$$r(p) = \left(\sum_i \alpha_i p_i^{\sigma+1} \right)^{\frac{1}{\sigma+1}} \quad (2)$$

Second, the optimal outputs of the X_i at $L_x = 1$ are found by applying Shepard's lemma to (2):

$$X_i = \alpha_i p_i^\sigma \left(\sum_j \alpha_j p_j^{\sigma+1} \right)^{\frac{1}{\sigma+1} - 1} = \alpha_i p_i^\sigma \left(\sum_j \alpha_j p_j^{\sigma+1} \right)^{\frac{-\sigma}{\sigma+1}} = \alpha_i \left(\frac{p_i}{r(p)} \right)^\sigma \quad (3)$$

Relative production of X_1 and X_2 thus depend on prices and comparative advantage. The total output of X_i is given by (3) multiplied by the total labor allocated to production L_x .

$$X_i = \alpha_i (p_i / r(p))^\sigma L_x \quad (4)$$

It may help to interpret the α 's to note from (2) and (3) that, if all prices equal one, then α_i will give the value share of X_i in total output.

Let E^* be the level or "endowment" of world environmental quality at zero production. Pollution is modeled as a reduction in the endowment of good E and is proportional to the total aggregate input L_x in both countries. Input L can also be used for an abatement activity, such as planting trees which absorb CO_2 . Aggregate labor in each country is then divided between production and abatement: $\bar{L} = L_x + L_a$ where \bar{L} is effective labor supply (explained shortly).

$$\begin{aligned}
\text{Pollution} &= \text{Reduction in } E \text{ endowment} = L_x^n + L_x^s & (5) \\
\text{Abatement} &= \text{Addition to the } E \text{ endowment} = L_a^n + L_a^s
\end{aligned}$$

For any allocation of labor between production and abatement in the two countries, world environmental quality is then given by:

$$E = E^* - (L_x^n + L_x^s) + (L_a^n + L_a^s) \quad (6)$$

Abatement is financed by consumption taxes in n and s: t^n, t^s . The government uses tax revenue to hire labor away from production and pays it to plant trees. Thus public policy, via the consumption tax, determines the allocation of the composite input L between production and abatement. Specifically, the equal taxes on both goods is equivalent to a tax t on the labor input to production. Budget balance then requires tax revenues $t w L_x$ to equal abatement expenditures $w L_a$ where w is the wage rate. Thus the tax rate (in either country) will equal the ratio of labor in abatement to labor in production.

$$t = \frac{L_a}{L_x} \quad \text{or} \quad \frac{t}{1+t} = \frac{L_a}{\bar{L}} = \text{share of labor in abatement} \quad (L_x = \bar{L} - L_a) \quad (7)$$

The advantage of this simple model is that it implies “neutrality” in several senses. By neutrality I mean:

- => no pollution-intensive sector
- => no comparative advantage in polluting sector
- => no factor-intensity, factor-endowment issues
- => no pollution-from-consumption-versus-production issue

The dominant model in the trade-and-environment literature has only one sector that pollutes, This leads to policy results that are very sensitive to:

- => which factor is intensive in which good
- => which good is the country’s comparative advantage good
- => whether pollution is from consumption or production

Here we avoid these issues. However, at the same time, allowing the α_i ’s to differ across countries generates a comparative advantage motive for trade and gains from trade in the X ’s. This will be important since world prices will change as one country withdraws labor from production in order to increase abatement. This terms-of-trade change in the relative prices of the two X goods will always favor the environmentally conscious country, a point I will return to shortly.

Preferences are Stone-Geary, lower case letters for per-capita or “household” quantities.

Household utility in either country is given by

$$u = x_1^\varepsilon x_2^\varepsilon (E + e_0)^\gamma \quad 2\varepsilon + \gamma = 1 \quad (8)$$

where e_0 is a positive parameter for each household in each country which creates the non-homotheticity: up to a critical level of income, there will be no demand for (or not willingness to pay for) additional environmental quality. It can be useful to think of e_0 as an endowment good given to each household which cannot be traded: every household can watch the sunset and that is a perfect substitute for a cleaner environment. E is the (world) environmental good supply given in (6) and is a pure (non-rival and non-excludable) public good. So each consumer in each country gets to consume the entire world supply.

Let E^i denotes the demand for environment in country i ,
 m^i is country i 's per-capita (household) income or labor productivity
 h^i is the number of households in country i
 $\bar{L}^i = m^i h^i$ is the total effective labor supply in i , equal to productivity per household time the number of households. L is used as numeraire, so country i 's total income is given by $M^i = m^i h^i = \bar{L}^i$
 p_e^i is the price (willingness to pay) by a single household h for environment in i .

Consumer (household) optimization yields:

$$E^i = \max \left[0, (\gamma - 1) e_0^i + \frac{\gamma m^i}{p_e^i} \right] \quad (9)$$

$$E^i > 0 \quad \text{iff} \quad m^i > \frac{(1 - \gamma)}{\gamma} p_e^i e_0^i \equiv m^0 \quad (10)$$

The result is the one just noted: up to the threshold per-capita income given in (10), there is no demand for environmental quality. Once the threshold income is reached,

$$E^i = (\gamma - 1) e_0^i + \gamma \frac{m^i}{p_e^i} \quad (\text{note that } (\gamma - 1) < 0) \quad (11)$$

At a constant price, the income elasticity of demand for E is greater than one once the threshold is reached. In equilibrium, E must be the same for all consumers in all countries (perfect global public good). So for (11) to hold, it must be that the willingness to pay p_e differs across

countries. This (private) willingness to pay can be found by inverting (11).²

$$p_e^i = \frac{\gamma m^i}{E + (1 - \gamma)e_0} \quad (12)$$

Since the environment is a public good (non-rival and non-excludable), optimal policy depends on the aggregate benefits of the good across households. The “planner’s problem” is to maximize the aggregate equivalent of (8): $U^i = (X_1^i)^\epsilon (X_2^i)^\epsilon (E + e_0 h^i)^\gamma$, where $X_j^i = x_j^i h^i$. The social-welfare maximizing level of E , analogous to (11), is given by

$$E^i = (\gamma - 1)e_0 h^i + \gamma \frac{m^i}{q_e^i} h^i \quad \text{where } m^i h^i = M^i \text{ is } i\text{'s total income} \quad (13)$$

When this is set equal to the (exogenous) quantity of E , the price that produces equality is denoted by q_e^i , which I’ll term the “social willingness to pay”. Invert (13).

$$q_e^i = \frac{\gamma m^i h^i}{E + (1 - \gamma)e_0 h^i} = \frac{\gamma M^i}{E + (1 - \gamma)e_0 h^i} \quad (14)$$

The income elasticity of social willingness to pay (for a fixed E) with respect to m^i , holding h^i constant is one, while the income elasticity with respect to h^i holding m^i constant is less than one.

Holding total income M^i constant, an *increase* in per-capita income is a *reduction* in the number of households h^i , and so the social willingness to pay rises. Note that this passes one simple check: if preferences are homothetic such that $e_0 = 0$, then the social willingness to pay depends only on total income and not per-capita income and the value of demand is a constant share of total income: $q_e^i E = \gamma M^i$. Note also that with or without homotheticity, the social price in (14) is greater than the private willingness to pay in (12) for $h > 1$.

From (14), the ratio of the social willingness to pay in n and s is given by

$$\frac{q_e^n}{q_e^s} = \left[\frac{E + (1 - \gamma)e_0 h^s}{E + (1 - \gamma)e_0 h^n} \right] \frac{M^n}{M^s} \quad (15)$$

Since E is common to the two countries, we get the following results:

²Note from (11) and (12) that the income elasticity of demand for E (greater than one), is greater than the income elasticity of willingness to pay (equal to one in (12)). This difference is explored in Flores and Carson (1997).

Result: country n will have the higher willingness to pay if:

- (a) both countries have the same total income but country n has a higher per-capita income ($M^n = M^s$, $h^n < h^s$).
- (b) both countries have the same number of households, but n has a higher per-capita income and therefore higher total income ($M^n > M^s$, $h^n = h^s$).
- (c) both countries have the same per-capita income, but country n has more households and therefore higher total income ($h^n = \rho h^s$, $M^n = \rho M^s$, $\rho > 1$).

Willingness to pay does not easily translate into an optimal tax rate (abatement effort) in this model. The introduction of this tax by one country will lead to an increase in E for both countries, lowering the willingness to pay for environment by the passive country: the q 's are endogenous to the tax. Thus non-cooperative and cooperative tax rates will differ. Secondly, an optimal tax must take into account the effect of withdrawing resources from production to use in abatement on the terms of trade in goods. This terms-of-trade effect can partially compensate country n for the lower production of goods when it uses resources for abatement, but it creates a corresponding loss for country s .

In order to understand the terms-of-trade effect, consider some restrictive assumptions in order to derive a clear expression for the effect of abatement by one country on the terms of trade. First, assume that the elasticity on transformation is zero, so that goods are produced in fixed proportions in each country, with that proportion depending on the country's alpha parameters. Assume that there is free trade in goods, so that the world production ratio equals the (inverse) consumer price ratio in equilibrium (since the goods are Cobb-Douglas symmetric substitutes in consumption). We then have:

$$\frac{X_1^n + X_1^s}{X_2^n + X_2^s} = \frac{\alpha_1^n L_x^n + \alpha_1^s L_x^s}{\alpha_2^n L_x^n + \alpha_2^s L_x^s} = \frac{p_2}{p_1} \quad (16)$$

Let country n withdraw a unit of labor from production. Differentiating (16), we have:

$$\frac{(\alpha_2^n L_x^n + \alpha_2^s L_x^s) \alpha_1^n - (\alpha_1^n L_x^n + \alpha_1^s L_x^s) \alpha_2^n}{(\alpha_2^n L_x^n + \alpha_2^s L_x^s)^2} dL_x^n = d(p_2/p_1) \quad (17)$$

where we will assume that country n has the comparative advantage in good 1: $\alpha_1^n > \alpha_2^n = (1 - \alpha_1^n)$ and $\alpha_2^s > \alpha_1^s = (1 - \alpha_2^s)$.

Assume that the countries are identical initially and that there are no initial abatement taxes, so that $L_x^n = L_x^s = L_x$ where L_x is the total endowment of each country. Second, assume

symmetry in the comparative advantage parameters such that $\alpha_1^n = \alpha_2^s = (1 - \alpha_2^n) = (1 - \alpha_1^s)$.

Because $\alpha_1^n + \alpha_2^n = 1$ and $p_1/p_2 = 1$ (symmetry in production and consumption), (17) simplifies to (shown in the appendix)

$$(\alpha_1^n - \alpha_2^n) \frac{dL_x^n}{L_x^n} = \frac{d(p_2/p_1)}{(p_2/p_1)} \quad \text{or} \quad -\frac{d(p_1/p_2)}{(p_1/p_2)} = \frac{d(p_2/p_1)}{(p_2/p_1)} = (\alpha_1^n - \alpha_2^n) \frac{dL_x^n}{L_x^n} \quad (18)$$

The effect of country n withdrawing a unit of labor from production ($dL_x^n < 0$) is to reduce the relative price of country s 's export good X_2 (country s has a deterioration in its terms of trade) with the size of this effect proportional to the comparative advantage spread $(\alpha_1^i - \alpha_2^i)$.

3. Analytical solutions for special cases

Continue to assume that $\bar{L} = m^i h^i$ is the same in both countries and the production ratio in each country is fixed as we have just done. But one country (n) may have fewer numbers of more productive households. Two special cases are instructive.

$(\alpha_1^n - \alpha_2^n) = 0$: no comparative advantage and hence no trade in goods. Country s has no adverse terms-of-trade effect and benefits from the reduction in admissions from country n .

This is shown in Figure 2a, where the (right angle) production frontiers are initially the same and country n 's frontier moves in with labor transferred to abatement. All the reduction in commodity consumption is borne by country n while both countries benefit from an increase in E .

$(\alpha_1^n - \alpha_2^n) = 1$, that is, $\alpha_2^n = \alpha_1^s = 0$: each country is fully specialized. Country s has the maximum possible adverse terms-of-trade effect but does benefit from the reduction in emissions from country n .

This is shown in Figure 2b: because of Cobb-Douglas preferences between X_1 and X_2 , the world price ratio changes so that both countries share the same reduction in commodity consumption. Fully half the burden of withdrawing resources from production by country n is shifted to country s . With Cobb-Douglas demand, each country will have the same income M :

$$M^n = p_1 X_1^n = p_1 L_x^n = M^s = p_2 X_2^s = p_2 L_x^s \quad \text{since} \quad \frac{X_1^n}{X_2^s} = \frac{p_2}{p_1} \quad (19)$$

which follows from (8), and the symmetry of the alphas ($\alpha_1^n = \alpha_2^s = 1$).

With identical Cobb-Douglas sub-utility functions over X_1 and X_2 , the countries will have identical (X_1, X_2) consumption bundles regardless of the level of country n 's abatement effort. In other words, the change in the terms-of-trade change fully compensates country n (relative to country s) for its unilateral abatement effort. Both countries have an identical consumption loss and the same aggregate improvement in the environment. We would expect that the full-specialization case should lead to higher non-cooperative abatement efforts as each country passes part of the burden of its tax to its partner.

Consider first the case of no comparative advantage. From our earlier assumptions, utility of country n is given by

$$U = U(L_x^n/2, L_x^n/2, E^* - L_x^n + L_a^n - L_x^s + L_a^s + e_0 h^n) \quad (20)$$

where $-L_x^n + L_a^n = -(\bar{L} - 2L_a^n)$. Using the explicit function form in (8) and exploiting the symmetry between the two consumption goods, this becomes

$$U = \left(\frac{\bar{L} - L_a^n}{2} \right)^{2\varepsilon} \left(E^* - (\bar{L} - 2L_a^n) - (\bar{L} - 2L_a^s) + e_0 h^n \right)^\gamma \quad (21)$$

The non-cooperative (Nash) solution is found by taking the derivative of (21) with respect to L_a^n holding L_a^s constant. This derivative rearranges to:

$$\frac{\varepsilon}{2\gamma} \geq \frac{(\bar{L} - L_a^n)/2}{E^* - 2\bar{L} + 2L_a^n + 2L_a^s + e_0 h^n} \quad \text{complementary } (\perp) \text{ to } L_a^n \geq 0 \quad (22)$$

If the first-order condition holds with strict equality, then the complementary variable is strictly positive and vice versa.

Give equal weights to all three goods, such that $\varepsilon = \gamma = 1/3$. (22) simplifies to

$$L_a^n \geq \bar{L} - E^*/3 - (2/3)L_a^s - (1/3)e_0 h^n \quad \perp \quad L_a^n \geq 0 \quad (23)$$

Let $E^* = 0$ to make things even simpler. Then divide through (23) by \bar{L} .

$$\frac{L_a^n}{\bar{L}} \geq 1 - \frac{e_0}{3m^n} - \frac{2}{3} \frac{L_a^s}{\bar{L}} \quad \perp \quad L_a^n \geq 0 \quad \text{since} \quad \frac{h^i}{\bar{L}} = \frac{1}{m^i} \quad (24)$$

Although I will use a consumption tax in the general-equilibrium model, a very simple tax is just one on all labor. With L as numeraire, the budget-balance condition that tax revenues equal payments to labor in abatement is given by $t^i \bar{L} = L_a^i$ or $t^i = L_a^i / \bar{L}$. (24) and the corresponding equation for country s become

$$t^n \geq 1 - \frac{e_0}{3m^n} - \frac{2}{3} t^s \quad \perp \quad t^n \geq 0 \quad t^s \geq 1 - \frac{e_0}{3m^s} - \frac{2}{3} t^n \quad \perp \quad t^s \geq 0 \quad (25)$$

These are Nash best-response functions giving each country's "optimal" (non-cooperative) tax as a function of its rival's tax. Clearly, the taxes are strategic substitutes. An increase in the North's tax improves the environment, therefore reducing the South's marginal willingness to pay, and therefore reducing the South's tax. Each country's best response tax is increasing in its per-capita income.

Assuming that the two equations have an interior solution (both taxes positive), the solution is:

$$t^n = \frac{3}{5} - \frac{3}{5} \frac{e_0}{m^n} + \frac{2}{5} \frac{e_0}{m^s} \quad t^s = \frac{3}{5} - \frac{3}{5} \frac{e_0}{m^s} + \frac{2}{5} \frac{e_0}{m^n} \quad (26)$$

Each country's tax is increasing in its per-capita income and decreasing in the other country's per-capita income, the two country's total incomes constant by assumption. The difference between the two countries' taxes is given by:

$$t^n - t^s = \frac{e_0}{m^n m^s} (m^n - m^s) \quad (27)$$

The symmetric solution when both countries have the same per-capita incomes is

$$t^n = t^s = \frac{3}{5} - \frac{e_0}{5m} \quad \text{at} \quad m = m^n = m^s \quad \text{critical } m: \quad m = \frac{e_0}{3} \quad (28)$$

where the critical m is the common value of m at which (28) holds with equality at $t = 0$. Taxes will be zero at any lower level of m . Once the tax kicks in at $m = e_0/3$, further increases in m , raise the tax rate, and so the share of resources devoted to abatement.

Now consider the case of full specialization, with country n producing X_1 and s producing X_2 , continuing to assume a common value of \bar{L} . The situation follows from equation (19) and Figure 1b. Each country's consumption will be half the world output of each good, so the identical consumption bundles are: $X_1^i = L_x^n/2$ and $X_2^i = L_x^s/2$. Utility in the North is:

$$U^n = U(L_x^n/2, L_x^s/2, E^* - L_x^n + L_a^n - L_x^s + L_a^s + e_0 h^n) \quad (29)$$

where $-L_x^n + L_a^n = -(\bar{L} - 2L_a^n)$. Notice in (29) that n 's consumption of X_2 is now not reduced by its own abatement effort. As we did above, (29) can be written as

$$U = \left(\frac{\bar{L} - L_a^n}{2} \right)^\varepsilon \left(\frac{\bar{L} - L_a^s}{2} \right)^\varepsilon \left(E^* - (\bar{L} - 2L_a^n) - (\bar{L} - 2L_a^s) + e_0 n^n \right)^\gamma \quad (30)$$

With the first-order condition given by

$$\frac{\varepsilon/2}{2\gamma} \geq \frac{(\bar{L} - L_a^n)/2}{E^* - 2\bar{L} + 2L_a^n + 2L_a^s + e_0 h^n} \quad \perp \quad L_a^n \geq 0 \quad (31)$$

Assuming again equally consumption weights such that that $\varepsilon = \gamma = 1/3$, we have

$$L_e^n \geq \bar{L} - E^*/2 - (1/2)L_a^s - (1/4)e_0 h^n \quad \perp \quad L_a^n \geq 0 \quad (32)$$

Divide through by \bar{L} as before and use $h^i/\bar{L} = 1/m^i$ and $t^i = L_a^i/\bar{L}$. (32) yields the best-response functions

$$t^n \geq 1 - \frac{e_0}{4m^n} - \frac{1}{2}t^s \quad \perp \quad t^s \geq 0 \quad t^s \geq 1 - \frac{e_0}{4m^s} - \frac{1}{2}t^n \quad \perp \quad t^s \geq 0 \quad (33)$$

Assuming an interior solution, we have

$$t^n = \frac{2}{3} - \frac{1}{3} \frac{e_0}{m^n} + \frac{1}{6} \frac{e_0}{m^s} \quad t^s = \frac{2}{3} - \frac{1}{3} \frac{e_0}{m^s} + \frac{1}{6} \frac{e_0}{m^n} \quad (34)$$

The difference between the tax rates in this solution is

$$t^n - t^s = \frac{e_0}{2m^n m^s} (m^n - m^s) \quad (35)$$

The symmetric solution when both countries have the same per-capita incomes is

$$t^n = t^s = \frac{2}{3} - \frac{e_0}{6m} \quad \text{at} \quad m = m^n = m^s \quad \text{critical } m: \quad m = \frac{e_0}{4} \quad (36)$$

Comparing (36) to (28), we see that non-cooperative taxes are higher under full specialization, where each country can pass on part of the burden of its tax to the other country. The critical m , the level of income at which a tax is first introduced, is lower in the case of full specialization (36).

The weak inequalities in (25) and in (33) each constitute two linear complementarity problems in the non-negative tax variables. Figure 3 shows a numerical example of these two cases, in which $e_0 = 2$. Moving to the right on the horizontal axis, per-capita income increases for the North, with the South's income being the reciprocal of the North's. The taxes diverge under growing income inequality and the taxes under full specialization are greater than under no comparative advantage.

Now let's consider a cooperative outcome or a Nash bargaining outcome using the non-cooperative outcome as a disagreement point. Impose the assumptions that both countries are the same size (\bar{L} is the same) and have the same per-capita income ($m^n = m^s$, $h^n = h^s$). Since everything is symmetric in this special case (including equal taxes in the non-cooperative outcome), the cooperative outcome will involve a common abatement effort $L_a = L_a^n = L_a^s$, and the solution can be represented by maximizing the utility of either country under the assumption of equal abatement efforts. Second, the solution will be the same in either the no-comparative-advantage case or the full-specialization case. In the latter case, each country produces $(\bar{L} - L_a)$ of its specialization good and trades half of that for the other country's good, consuming $(\bar{L} - L_a)/2$ of each good. In the no-comparative-advantage case, each country both produces and consumes $(\bar{L} - L_a)/2$ of each good. So both cases will yield the same cooperative outcome given the symmetry assumptions. Dropping the country superscript on L_a and h in (21), utility for each country is maximized with respect to a common value of L_a . Equation (21) becomes

$$U = \left(\frac{\bar{L} - L_a}{2} \right)^{2\varepsilon} (E^* - 2\bar{L} + 4L_a + e_0 h)^\gamma \quad (37)$$

The cooperative and Nash-bargaining solution is found by taking the derivative of (37) with respect to L_a . This derivative rearranges to:

$$\frac{\varepsilon}{4\gamma} \geq \frac{(\bar{L} - L_a)/2}{E^* - 2\bar{L} + 4L_a + e_0 h^n} \quad \text{complementary } (\perp) \text{ to } L_a \geq 0 \quad (38)$$

Give equal weights to all three goods, such that $\varepsilon = \gamma = 1/3$. (38) simplifies to

$$L_a \geq (2/3)\bar{L} - E^*/6 - (1/6)e_0 h^n \quad \perp \quad L_a \geq 0 \quad (39)$$

Let $E^* = 0$ to make things even simpler. Then divide through (39) by \bar{L} and use $h/\bar{L} = 1/m$ and $L_a/\bar{L} = t$ as before.

$$t^n = t^s = \frac{2}{3} - \frac{e_0}{6m} \quad \text{at} \quad m = m^n = m^s \quad (40)$$

Here we have an interesting result that the cooperative outcome is the same as the non-cooperative outcome under the full-specialization assumption (equation (36)). Under the latter, each country raises its tax non-cooperatively because it can pass on half the burden to the other country. Under all our special symmetry assumptions, this substitutes perfectly for a country raising its tax in order to cooperatively internalize its externality on the other country.

Two final comments before moving on to general equilibrium. First, it is probably clear that an economy can exhibit a non-monotonic Kuznets' curve. Starting from a very low m , the optimal taxes are zero as income increases, meaning that there is an unambiguous fall in E . Eventually taxes kick in, and after that the tax rates rise with m , and so the share of labor devoted to abatement rises. At some point, environmental quality may exceed the initial quality, an outcome we will see in the next section. Second, note from the best-response functions (26) and (33) that we can have something I dub "policy leakage". As one country increases its abatement tax and effort, the environment improves and so the optimal response of the other country is to lower its tax. This is reinforced to the extent that the increase in the first country's tax makes the second country poorer through the terms-of-trade effect and hence lowers its willingness to pay.

4. The general-equilibrium simulation model

The model developed above seems simple, but there is a lot more simultaneity if relative commodity outputs are endogenous and countries may differ in size (L) as well as per-capita incomes. Optimal policy depends on income, for example, and income depends on the policy chosen, both by determining the domestic resources available for production of goods and through the international general-equilibrium terms-of-trade effect. Note, for example, that the simple result in (18) requires severe assumptions and even then it is only locally valid in the

neighborhood of zero abatement. So let's turn to a numerical general-equilibrium model to see how per-capita income matters for cooperative and non-cooperative outcomes.

The model belongs to a class of problems loosely known as MPEC: mathematical programming with equilibrium constraints. In our case, the set of equilibrium constraints is the two-country general-equilibrium model. The latter, in turn, is known as an MCP: mixed complementarity problem. This is a set of weak inequalities with associated non-negative variables such as quantities and prices. When a weak inequality holds as an equality, the complementary variable is positive, zero if the inequality is strict in equilibrium.

The MPEC consists of maximizing some function such as a Nash bargaining function (cooperative) with respect to instrument variables such as tax rates (pollution taxes and/or tariffs) subject to the economic equilibrium constraint set. A non-cooperative Nash equilibrium is found by iteration: maximize the welfare of i holding j 's taxes constant, then hold i 's taxes constant at the solution values and maximize j 's welfare, repeat. This converges to a best-response, non-cooperative outcome in about eight iterations. There is one small difference from the model above that has no effect on any results.³ Here is a description of the model.

(A) Alternative objective functions

welfare of country n
welfare of country s
joint welfare (or Nash bargaining function)

(B) The mxm economic equilibrium problem (constraint set): 28 inequalities and unknowns

Inequalities	Complementary Variables	Number
marginal cost \geq price	quantities	
production of Z by n, s	quantities of Z	2
trade in X_1, X_2 by n, s	quantities traded	4
welfare in n and s	welfare in n and s	2
abatement activities in n and s	quantity of abatement	2
market clearing: supply \geq demand	prices	
supply / demand for Z^n, Z^s	prices of Z in n, s	2
supply / demand for X_i in n, s	prices of X_i in n, s	4
supply / demand for L in n and s	prices of L in n, s	2
supply / demand for welfare	price index in n, s	2

³I define a composite output quantity Z in each country equal to the left-hand side of (1), analogous to a Dixit-Stiglitz composite differentiated good in monopolistic-competition models. This carries a price p_z which can be useful in comparing the relative price of consumption to the environment.

supply / demand for abatement in n, s	price of abatement in n, s	2
supply / demand for environment good	willingness to pay in n, s	2
income balance: income \geq expenditure		
income balance for n, s	income in n, s	2
auxiliary equations		
pollution reduction = abatement	pollution abatement	1
pollution = emissions	pollution	1
(C) Additional unmatched instrument variables chosen to optimize welfare:		
pollution abatement effort (abatement tax) in countries n, s		2
tariffs imposed by countries n, s		2
(D) The MPEC (mathematical programming with equilibrium constraints)		
Maximize objective function, subject to:		
instruments: pollution abatement taxes and tariffs in n, s		4
mxm economic equilibrium problem constraint set		28

A sample program, written and solved in GAMS, is provided in appendix two to the paper.

5. Policy experiments⁴

The first policy experiment is shown in Figure 4. This considers environmental quality as a function of per-capita income, where I make the countries identical (in total and in per-capita incomes) for simplicity. Productivity or “effective” labor units per household are increased holding the number of households constant. The MPEC solves for the optimal taxes or abatement effort at each level of income (productivity).

This produces a non-monotonic Kuznets curve in Figure 4 as we expect. At very low levels of per-capita income, there is no demand for abatement or additional environmental quality and the latter falls with increases in productivity. At a critical level around 0.9 in this

⁴Somewhat different parameterizations are used here than in simplified analytical model. Here environmental quality is: $E = E^* - (L_x^n + L_x^s)/2 + 4(L_a^n + L_a^s)$ with $E^* = (\bar{L}^n + \bar{L}^s)/2$. A consumption tax is used instead of a labor tax, though there is a simple relationship between them as can be seen from (10): the equivalent labor tax is equal to $t/(1+t)$ where t is the consumption tax. These are simply for expositional purposes and do not affect the qualitative results in any way.

experiment, there is a positive demand for additional environmental quality and the tax kicks in. The tax *rate* rises steadily thereafter due to the non-homotheticity, and is equal to 0.25 on the right-hand boundary where environmental quality is now higher than in the very poor county.

What is perhaps not so obvious is that there is still some non-monotonicity in the Kuznets curve with homothetic demand. This is due to the fact that the initial fixed endowment of $E = E^*$ is too high or rather the demand price too low to justify abatement at low income levels. Intuitively, a really poor country would want to sell off some of the environmental good if it could under either homothetic or non-homothetic demand. Adding the non-homotheticity assumption shifts the minimum point to the right in Figure 4 (not shown). If there was no initial endowment of E , $E^* = 0$, then the homothetic case would produce a constant tax rate at all levels of income such that environmental quality is a linear function of income passing through the origin, while the non-homothetic case continues to look like Figure 4 (E can go negative provided that $-E < e_0$). This possibly expected result occurs only when $E^* = 0$.

Figure 5 presents simulation results that relate to three issues that have received a lot of attention in the trade/environment literature. In these simulations, both countries have the same total income (L) but country n has a per-capita income 1.5 times that of country s (n has fewer, more productive households). There is a strong pattern of comparative advantage in the case considered, $\alpha_1^n = \alpha_2^s = 0.9$, and $\sigma = 1$. “Leakage” in the literature typically works through price changes when sectors differ in pollution intensities. A carbon tax in one country pushes down the world price of fuels and pushes up the world price of the carbon-intensive sector, thus transferring production and fuel use to the other country. In my model, there is no pollution intensive sector, but there leakage through abatement changes as I briefly mentioned earlier. The left-hand panel of Figure 5 graphs the effect of an exogenous increase in country n ’s abatement tax on the abatement efforts of both countries. As country n ’s tax and effort increase, the best response of country s is to decrease its tax and abatement for two complementary reasons: first, the environment is cleaner (E rises) and second, the terms-of-trade effect makes country s poorer.

The right-hand panel of Figure 5 relates to a literature on “border-tax adjustments”, in which a country may impose import tariffs on the carbon content of imports. In the figure, an (exogenous - not optimized) tariff for country n is graphed on the horizontal axis. The effect of the tariff is to make country s poorer, and country s ’s optimal response is to lower its abatement effort. A border-tax adjustment is counter-productive. Furthermore, the tariff reduces trade volume and country n is less able to pass on part of its abatement costs by changes in the terms of trade. For the particular parameter values in this simulation, country n actually responds to its own (exogenous) tariff by reducing its own abatement effort.

The bottom panel of Figure 5 addresses a very old but very important question: is free-trade good for the environment, with an answer here in the affirmative. I show a simple case where the two countries are identical, and they have identical, *exogenous* import tariffs (late these are endogenous). The vertical axis gives the *endogenous* abatement taxes (and thus abatement efforts) of the two countries as a function of their common tariff level in non-

cooperative abatement setting. Protection is bad for the environment because (a) it makes the countries poorer and thus lowers their abatement efforts. (b) higher tariffs make it less possible to pass on part of a country's abatement costs to the other country (applies to the non-cooperative case only).

The next exercise is to examine how cooperative and non-cooperative policy outcomes depend on per-capita income. Some results are shown in Figure 6. As in Figure 5, the total incomes (\bar{L}) of the countries are equal and there is a strong pattern of comparative advantage: $\alpha_1^n = \alpha_2^s = 0.9$, and $\sigma = 1$. Thus when per-capita incomes and taxes are unequal, there will be a fairly strong terms-of-trade effect that favors country n .

The solid boxes in Figure 6 show an outcome when the two countries have equal per-capita incomes (or same number of equally productive households). Each country's (unilateral) optimal tax is shown when the other country's tax is zero as are the non-cooperative Nash equilibrium rates. There is no difference in the values of the taxes across countries in this latter equilibrium as we expect to be the case. The (equal) cooperative tax rates are Nash bargaining outcomes where the disagreement outcome is the non-cooperative Nash equilibrium. The cooperative rates are considerably higher than the non-cooperative ones as shown in Figure 6. The terms-of-trade in both outcomes is one and there is no trade in goods.

The second set of outcomes, shown with a slash through the boxes in Figure 6, assumes that country n has a per-capita income 1.5 times that of country s . Total incomes are the same as in the first case (solid boxes), so country n now has fewer, more productive households while s has more, less productive households. Figure 6 shows that the non-cooperative equilibrium shifts to a higher tax for country n and to a lower tax for country s (as we would expect). The cooperative Nash bargaining outcome using the non-cooperative outcomes as the disagreement point shifts from the equal-per-capita-income scenario in about the same way.

Table 1 presents numerical values for these results and compares them to several alternative scenarios. The first two columns of numbers are the case where country n 's per-capita income is 1.5 times that of country s as in Figure 6. The first row gives the welfare values when there is no intervention by either country, where these welfare values are normalized at one. The second and third rows of Table 1 give the welfare values for the non-cooperative and cooperative outcomes, and the lower panel the corresponding tax rates ($PTAXN$, $PTAXS$), which are those in Figure 6. Note that the non-cooperative outcome results in a substantial welfare gain over non-intervention: unlike a non-cooperative tariff "war" for example, here the non-cooperation is a failure to internalize a positive rather than a negative externality.

The tax rates that maximize the Cobb-Douglas world welfare function and a Rawlsian welfare function are also shown (world welfare is the minimum of the two countries' welfare levels. In the fourth and fifth rows of Table 1 (and tax rates below)). The Cobb-Douglas is equivalent to a Nash bargaining outcome when the disagreement outcome for both countries is zero

An interesting feature of these results is that country n is actually the relative gainer over the no-intervention outcomes (except the Rawlsian one). How much of this is simply due to the fact that country n places a much higher value on the environment at the no-intervention point, and how much might be due to the this terms-of-trade effect? In order to examine this question, I compute an alternative scenario in which there is no comparative advantage: $\alpha_1^n = \alpha_2^s = 0.5$.

Results are shown in columns 3 and 4 of Table 1. Here we see some significant differences. First, the relative gainer is reversed in the cooperative and non-cooperative Nash outcomes. Now country s is the relative gainer. This verifies the conjecture that the terms-of-trade effect that favors the higher-tax country n in columns 1 and 2 is indeed important in determining the relatively larger gains for country n in those columns. Second, note in the lower part of the Table that the non-cooperative rates for both countries are lower when there is no comparative advantage. This is clearly due to the fact that raising your tax rate has no compensating beneficial effect on the terms of trade with no comparative advantage. Third, the cooperative Nash is almost unchanged from the comparative-advantage case, which seems to follow from section 3: cooperative internalizes the terms-of-trade externality to the extent there is one.

The CD maximum rates are also unchanged in alternative scenario 1, because there is no terms-of-trade effect when the countries have the same rates. Fourth, the Rawlsian maximum rate is now lower for country n and higher for country s . I think that the intuition here is that, starting from the Rawlsian taxes with comparative advantage, removing comparative advantage reduces the welfare of high-tax country n and raises it for country s . The Rawlsian outcome then adjusts n 's tax down and s 's tax up in alternative scenario 1.

A second alternative case is presented in columns 5 and 6 of Table 1. Comparative advantage is reinstated, but country n now has ten times the per-capita income of country s . Results are now that country s has a zero tax rate at both the cooperative and non-cooperative Nash outcomes. The non-cooperative outcome is in fact a Pareto optimum: there are no gains from cooperative. Second, note that country s is indifferent between the no-intervention outcome and the cooperative or non-cooperative equilibrium (in fact, I search until I found the per-capita income difference, a factor of ten, that gave this borderline result). If I push the size difference a little higher, then country s is actually worse off than with no intervention. Country s places little value on improved environmental quality and suffers a negative terms-of-trade effect when country n imposes its abatement tax. The relative price of country n 's export good is 1.22 at the cooperative and non-cooperative tax rate of 0.41 in alternative scenario 2.

Alternative case 2 also gives large difference between CD world welfare maximum tax rates and the Rawlsian rates. The CD rates are again equal to one another, while country n bears all the abatement effort in the Rawlsian equilibrium, which is equal to the non-cooperative Nash equilibrium in all respects.

I'll mention one more result, though omit an analysis for brevity. The model can consider an experiment in which countries agree to a common tax rule $t^n = t^s$, and then bargain over the

level of this shared tax rate. When the difference between the countries' per-capita incomes is a factor of ten as in columns 5 and 6 of Table 1, country s cannot gain at any common tax rate. Thus having agreed on an "equal sharing rule" and then bargaining on the rate, the outcome is a zero rate when the per-capita-income difference is large. This has some clear applications to situations such as the Kyoto protocol, where there was considerable controversy over whether or not the poor and rich countries should suffer the same proportional cuts in carbon emissions. The result was that the poor countries did not participate at all, which is consistent with the model here.

Now I turn to issue linking and introduce two additional policy instruments: import tariffs for countries n and s . We will assume that country n has five times both the total income and the per-capita income of country s . Comparative advantage is the same as we used earlier: $\alpha_1^n = \alpha_2^s = 0.9$ and $\sigma = 1$. A shorted version of the GAMS program (it doesn't solve for all scenarios) is attached at the end of the paper. The no-intervention case shown in the first row of Table 2 fixes the four policy instruments at zero.

Table 2 then gives the non-cooperative Nash equilibrium in the second row, where each country jointly chooses its abatement tax and tariff for fixed values of the other country's instruments.⁵ This is solved as an iterative MPEC. Country n 's welfare is maximized with respect to $PTAXN$ and $TARN$ holding $PTAXS$ and $TARS$ constant. Then the solution values of $PTAXN$ and $TARN$ are held constant and the welfare of country s is maximized with respect to $PTAXS$ and $TARS$. The iteration converges to the non-cooperative Nash equilibrium in about six to eight iterations.

Table 2 shows that, not surprisingly, country n has a high abatement tax and tariff. Country s has a small tariff and a zero abatement tax. The tariff difference would be expected by any trade economist from simple non-cooperative tariff theory given the size difference. Table 2 notes a substantial welfare improvement over no intervention for both countries: country s benefits more from n 's abatement effort than s is hurt by n 's tariff. However, I imagine a case can easily be produced in which country s is worse off than in the no-intervention equilibrium.

Four cooperative bargaining outcomes are then computed using the Nash equilibrium as a disagreement point in Table 2. The first computes an isolated environmental tax bargain, tariffs held at their Nash values. The second computes an isolated tariff bargain, abatement taxes held at their Nash values. One interesting result here is that there are no gains to an isolated environment negotiation. No Pareto improving changes in taxes can help and nothing will come of such a negotiation (sound familiar?). The isolated tariff negotiation does produce a positive result, as country n lowers its tariff significantly in exchange for country s eliminating its tariff.

⁵The welfare numbers are normalized at 1.0 in the non-cooperative outcome in the second row, but I should note that the difference in the welfare levels is nowhere near a factor of five: n 's welfare is about twice that of country s . This is due to a large relative price advantage for country s : the relative price ratio is 0.472 or inverting, the relative price of country s 's export good X_2 is 2.121.

In the third case using the non-cooperative outcome as a disagreement point (row 5 of Table 2), country n offers a lower tariff in exchange for a positive abatement effort by country s . In my view, this is what some writers and politicians in high-income countries want to do: offer trade liberalization in exchange for environmental and labor standards to poor countries. Results in Table 2 indicate a Pareto improvement with the north trading trade liberalization for abatement by the south.

Row 6 of Table 2 takes the non-cooperative Nash outcome as a disagreement outcome and computes a cooperative bargaining solution treating all four instruments as endogenous variables. Here the outcome is a zero abatement tax and a zero tariff for country s . Country n bears the burden of abatement and, in exchange, retain a high tariff but substantially less than the non-cooperative tariff.

Row 6 emphasizes the fact that free trade may not be achievable even in the four-instrument case using the Nash equilibrium as the disagreement outcome. Row 7, the final one in Table 2, therefore adds a final instrument which is a transfer payment to the other four instruments. In this case, the outcome does involve free trade as intuition probably suggests. It requires a large transfer payment from s to n : in effect, s bribes n into free trade (this transfer result is driven by the trade distortion, not the environment distortion). Row 7 also indicates that the north should bear all of the abatement effort. This is being driven by country size: with s a lot smaller, the world is scarce in good X_2 , south's comparative-advantage good, and abundant in X_1 , north's comparative-advantage good. Thus efficiency dictates that the real cost of the abatement should fall on X_1 , meaning the north does the abatement.

It is interesting to see, in both Table 1 and Table 2, that cooperative bargaining does not extract much in the way of additional gains. The non-cooperative outcomes do a "good job" of extracting gains, since the non-internalized pollution externality is a positive spillover between countries. I have no reason to believe that this specific result has great generality beyond this specific case: results are suggestive and illustrative.

6. Summary and Conclusions

The purpose of the paper is to offer an alternative base-case model for trade and the environment. I move away from a focus on differing pollution intensities across sectors and Pigouvian taxes and tariffs. The focus is on per-capita income and a resource using abatement activity as determinants of cooperative and non-cooperative international trade policy. The setting is a world with a global environmental externality such as CO₂ pollution and consumers who have a high income-elasticity of demand for environmental quality. While some of the results are not surprising, there are a number of interesting subtleties than may have gone unnoticed.

A couple of basic results are that (a) environmental quality will exhibit a u-shape with respect to per-capita income, an example of an environmental Kuznets curve. (b) non-

cooperative outcomes will involve “policy leakage”: an increase in one country’s abatement effort improves the world environment and hence leads the other country to reduce its effort. (c) border taxes are unhelpful: they just make your trading partner poorer and reduce its abatement effort. (d) free trade is good for the environment because (1) it raises welfare and thus leads to a higher abatement effort and (2) it allows a country to pass on more of the costs of abatement to the other country (in the non-cooperative case), thus raising its abatement effort.

But the bulk of the paper focuses on countries with different per-capita incomes. I show that a poorer country will have a lower abatement effort in both a non-cooperative outcome and in a Nash bargaining outcome using the non-cooperative equilibrium as a disagreement outcome. It may be that the poor country makes no abatement effort in a non-cooperative outcome and, beginning from that equilibrium, it may be that there are no gains to bargaining when abatement taxes are the only instrument available to the countries.

I believe that the terms-of-trade externality and the focus on a “real” abatement activity rather than Pigouvian taxes is fairly novel, though Bogmans (2015) also has both these features: when a country withdraws resources from production to abate, it effectively shrinks its economy, and this will move the world terms of trade in its favor. Thus it passes on part of the cost of abatement to the non-abating country. I show a case using the simple analytical model where fully half the cost is passed on, and so it turns out that the non-cooperative and cooperative outcomes are the same.

The final section of the paper allows countries to set tariffs as well as abatement taxes, and focuses on the case where the rich country is large, and so has both a high tariff and a high abatement tax. Part of the advantage of the numerical model, using the relatively new MPEC solver in GAMS, is that we can compute a Nash equilibrium in four instruments. Then using this as a disagreement outcome, we can compute bargaining outcomes for linked or unlinked negotiations. Results are interesting, but my (superficial) knowledge of bargaining theory suggests that general results (e.g., is it better to link) are unlikely to be found.

In the area of understanding how we got to where we are, the results here may help explain why the high-income countries often seem to give up more than they get in international negotiations (some will surely dispute this assertion), such as the Kyoto Protocol. While this may indeed be all or in part due to simple altruism, some of our results here suggest that this is also predicted by standard economic theories of cooperative and non-cooperative behavior. Specifically, with environmental quality a high-income-elasticity good, standard theory predicts a high abatement effort by high income countries, no need to appeal to altruism.

APPENDIX

First, the algebra on the CET transformation function, the national product function and the supply functions. I derive the unit national product function: the revenue derived from one unit of labor input into production.

$$\text{Max}_x \sum p_i x_i + \lambda \left[\left(\sum_i \alpha_i \left(\frac{x_i}{\alpha_i} \right)^\beta \right)^{\frac{1}{\beta}} - 1 \right] \quad (\text{A1})$$

$$\frac{\partial L}{\partial X_i} = p_i - \lambda \frac{1}{\beta} \left(\sum_i \alpha_i \left(\frac{x_i}{\alpha_i} \right)^\beta \right)^{\frac{1}{\beta} - 1} \alpha_i \beta \left(\frac{x_i}{\alpha_i} \right)^{\beta - 1} \frac{1}{\alpha_i} = 0 \quad (\text{A2})$$

$$\frac{p_j}{p_i} = \left(\frac{x_i/\alpha_i}{x_j/\alpha_j} \right)^{\beta - 1} \quad \frac{x_j}{\alpha_j} = \left(\frac{p_j}{p_i} \right)^{\frac{1}{\beta - 1}} \frac{x_i}{\alpha_i} = \left(\frac{p_j}{p_i} \right)^\sigma \frac{x_i}{\alpha_i} \quad (\text{A3})$$

$$p_j x_j = \alpha_j p_j^{\sigma + 1} p_i^{-\sigma} x_i / \alpha_i \quad (\text{A4})$$

$$r(p) = \sum p_j x_j = \left(\sum \alpha_j p_j^{\sigma + 1} \right) p_i^{-\sigma} \frac{x_i}{\alpha_i} \quad (\text{A5})$$

Inverting this gives unit supply functions: the optimal output of each good for one unit of labor input into production.

$$x_i = \alpha_i p_i^\sigma \left(\sum \alpha_j p_j^{\sigma + 1} \right)^{-1} r(p) \quad (\text{A6})$$

$$\alpha_i \left(\frac{x_i}{\alpha_i} \right)^{\frac{\sigma + 1}{\sigma}} = \alpha_i p_i^{\sigma + 1} \left(\sum \alpha_j p_j^{\sigma + 1} \right)^{-\frac{\sigma + 1}{\sigma}} r(p)^{\frac{\sigma + 1}{\sigma}} \quad (\text{A7})$$

$$\sum \alpha_i \left(\frac{x_i}{\alpha_i} \right)^{\frac{\sigma + 1}{\sigma}} = \sum \left(\alpha_i p_i^{\sigma + 1} \right) \left(\sum \alpha_j p_j^{\sigma + 1} \right)^{-\frac{\sigma + 1}{\sigma}} r(p)^{\frac{\sigma + 1}{\sigma}} = \left(\sum \alpha_i p_i^{\sigma + 1} \right)^{-\frac{1}{\sigma}} r(p)^{\frac{\sigma + 1}{\sigma}} \quad (\text{A8})$$

$$\left(\sum \alpha_i \left(\frac{x_i}{\alpha_i} \right)^{\frac{\sigma+1}{\sigma}} \right)^{\frac{\sigma}{\sigma+1}} = 1 = \left(\sum \alpha_i p_i^{\sigma+1} \right)^{-\frac{1}{\sigma+1}} r(p) \quad (\text{A9})$$

This gives us the unit national product function, which could also be termed the producer price index in an analogy to the CES consumer price index so widely used in monopolistic competition and economic geography.

$$r(p) = \left(\sum \alpha_i p_i^{\sigma+1} \right)^{\frac{1}{\sigma+1}} = p_z \quad (\text{A10})$$

The unit national product function or producer price index is also CET, a self-dual property familiar from the CES demand literature. Substitute this into the supply functions above.

$$x_i = \alpha_i p_i^{\sigma} \left(\sum \alpha_j p_j^{\sigma+1} \right)^{-1} \left(\sum \alpha_i p_i^{\sigma+1} \right)^{\frac{1}{\sigma+1}} = \alpha_i p_i^{\sigma} \left(\sum \alpha_j p_j^{\sigma+1} \right)^{-\frac{\sigma}{\sigma+1}} \quad (\text{A11})$$

This gives us more compact unit and total supply functions.

$$x_i = \alpha_i p_i^{\sigma} r(p)^{-\sigma} \quad X_i = \alpha_i p_i^{\sigma} r(p)^{-\sigma} L_z \quad (\text{A12})$$

Second, the algebra for the terms-of-trade effect under the special assumptions noted in the text.

$$\frac{X_1^n + X_1^s}{X_2^n + X_2^s} = \frac{\alpha_1^n L_z^n + \alpha_1^s L_z^s}{\alpha_2^n L_z^n + \alpha_2^s L_z^s} = \frac{p_2}{p_1} \quad (\text{A13})$$

$$\frac{(\alpha_2^n L_z^n + \alpha_2^s L_z^s) \alpha_1^n - (\alpha_1^n L_z^n + \alpha_1^s L_z^s) \alpha_2^n}{(\alpha_2^n L_z^n + \alpha_2^s L_z^s)^2} dL_z^n = d(p_2/p_1) \quad (\text{A14})$$

Assume that the countries are identical initially and that there are no initial abatement taxes, so that $L_z^n = L_z^s = L_z$ where L_z is the total endowment of each country. Second, assume symmetry in the comparative advantage parameters such that $\alpha_1^n = \alpha_2^s = (1 - \alpha_2^n) = (1 - \alpha_1^s)$. Then (A14) can be simplified to

$$\frac{(\alpha_1^{n^2} - \alpha_2^{n^2}) L_z^n L_z^s}{(\alpha_1^n + \alpha_2^n)^2 L_z^{s^2}} \frac{dL_z^n}{L_z^n} = d(p_2/p_1) \quad (\text{A15})$$

Because $\alpha_1^n + \alpha_2^n = 1$ and $p_1/p_2 = 1$ initially, this further simplifies to

$$(\alpha_1^n - \alpha_2^n) \frac{dL_z^n}{L_z^n} = \frac{d(p_2/p_1)}{(p_2/p_1)} \quad \text{or} \quad \frac{d(p_2/p_1)}{(p_2/p_1)} = (\alpha_1^n - \alpha_2^n) \frac{dL_z^n}{L_z^n} \quad (\text{A16})$$

The effect of country n withdrawing a unit of labor from production ($dL_z^n < 0$) is to reduce the relative price of country s's export good (country s has a deterioration in its terms of trade) with the size of this effect being proportional to the size of comparative advantage spread.

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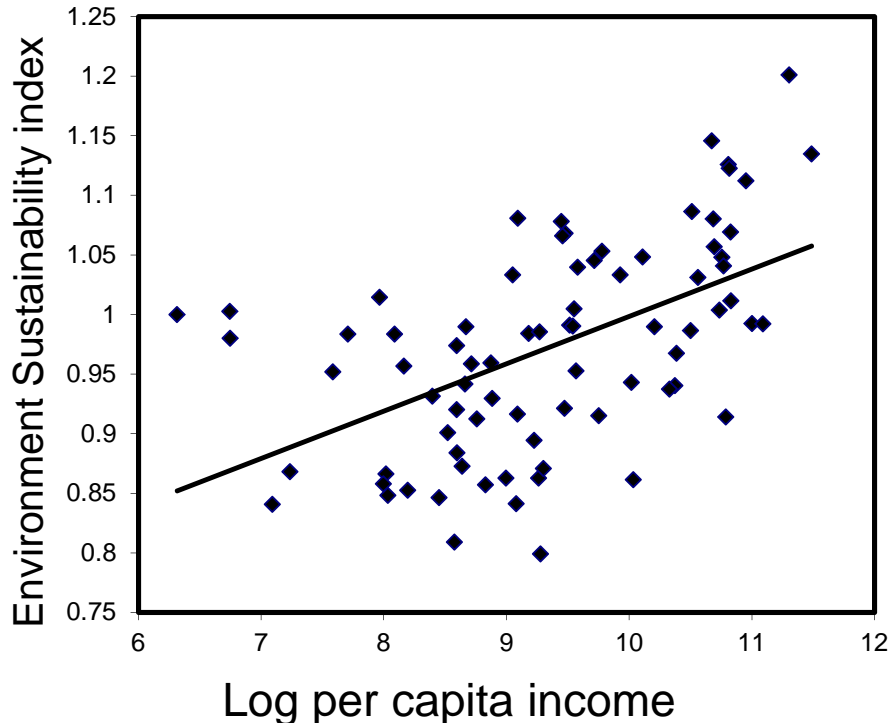
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Environmental sustainability index and per-capita income



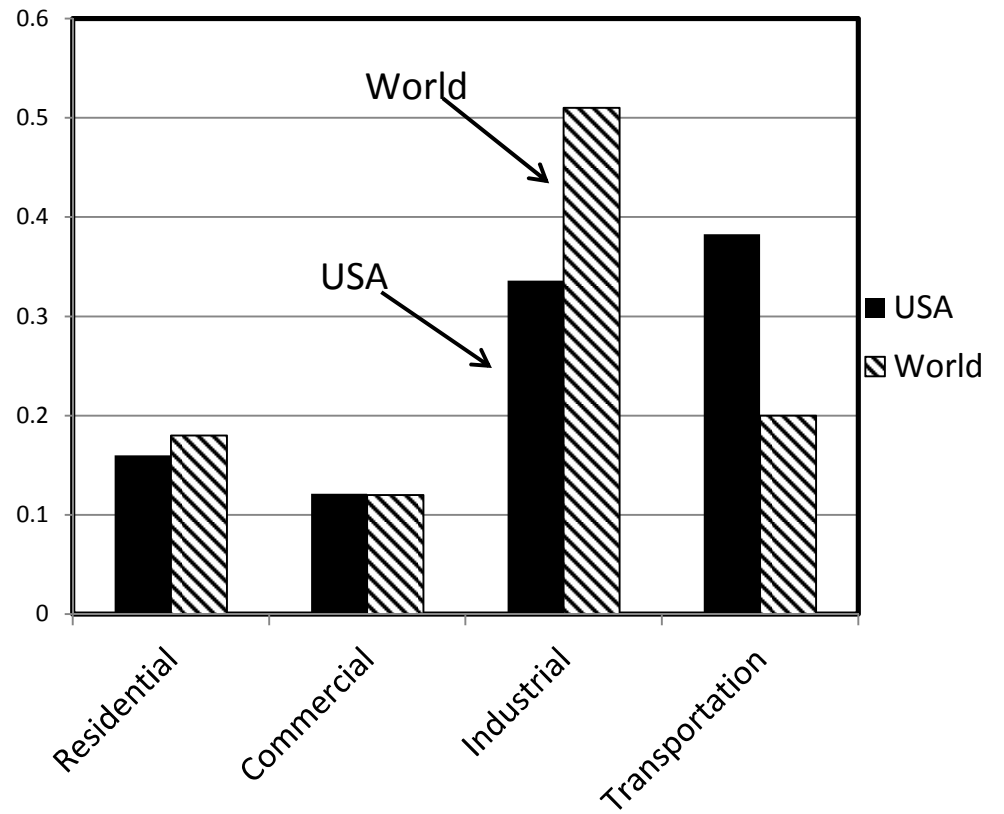
$$esi = 0.60 + 0.040 \cdot \ln(pci) \quad \text{adj } R^2 = 0.273$$

(8.84)
(5.51)

t-stats

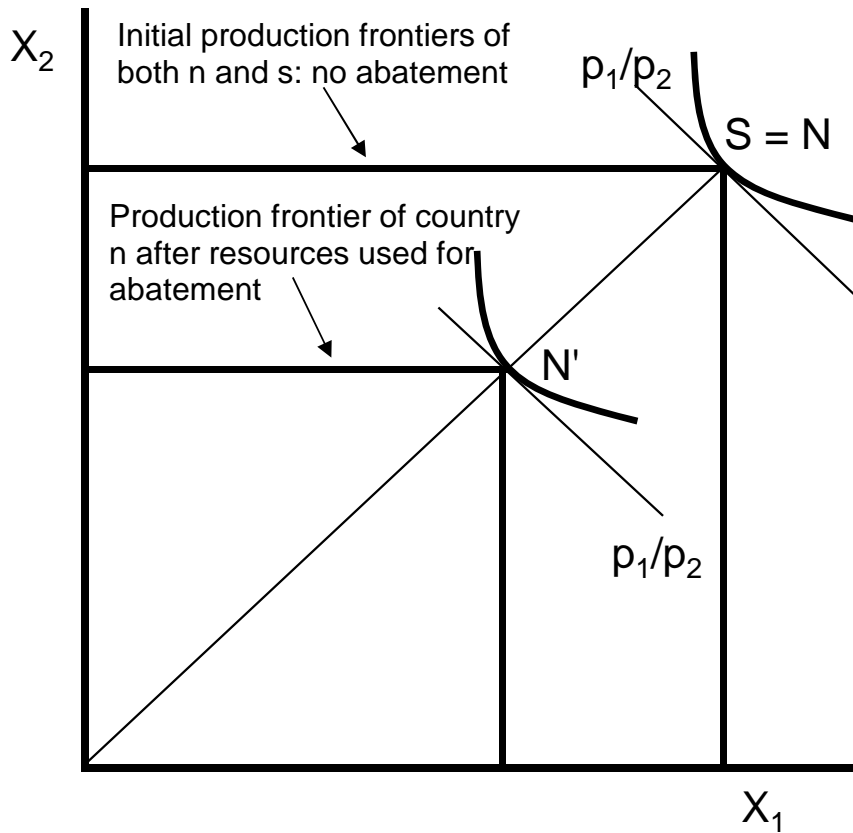
World Economic Forum, Global Competitiveness Report 2012-2013

Energy consumption by sector, USA and World, 2011



Energy Information Agency, Department of Energy
 US Government
<http://www.eia.gov/tools/faqs/faq.cfm?id=447&t=1>

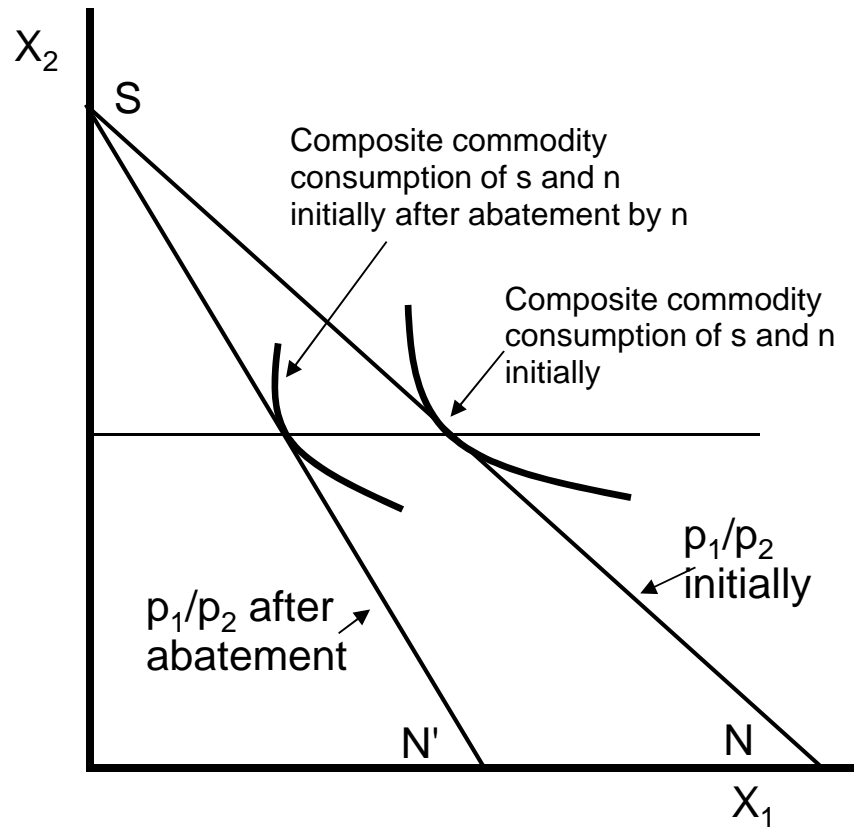
Figure 2a: Effect of abatement by country n on the commodity consumption of both countries: no comparative advantage



S = N: production and consumption of s and n initially

N': production and consumption of n after abatement

Figure 2b: Effect of abatement by country n on the commodity consumption of both countries: complete specialization



S: production point of country s

N: initial production point of country n before abatement

N': production point of country n after abatement

Figure 3: Abatement taxes and per-capita

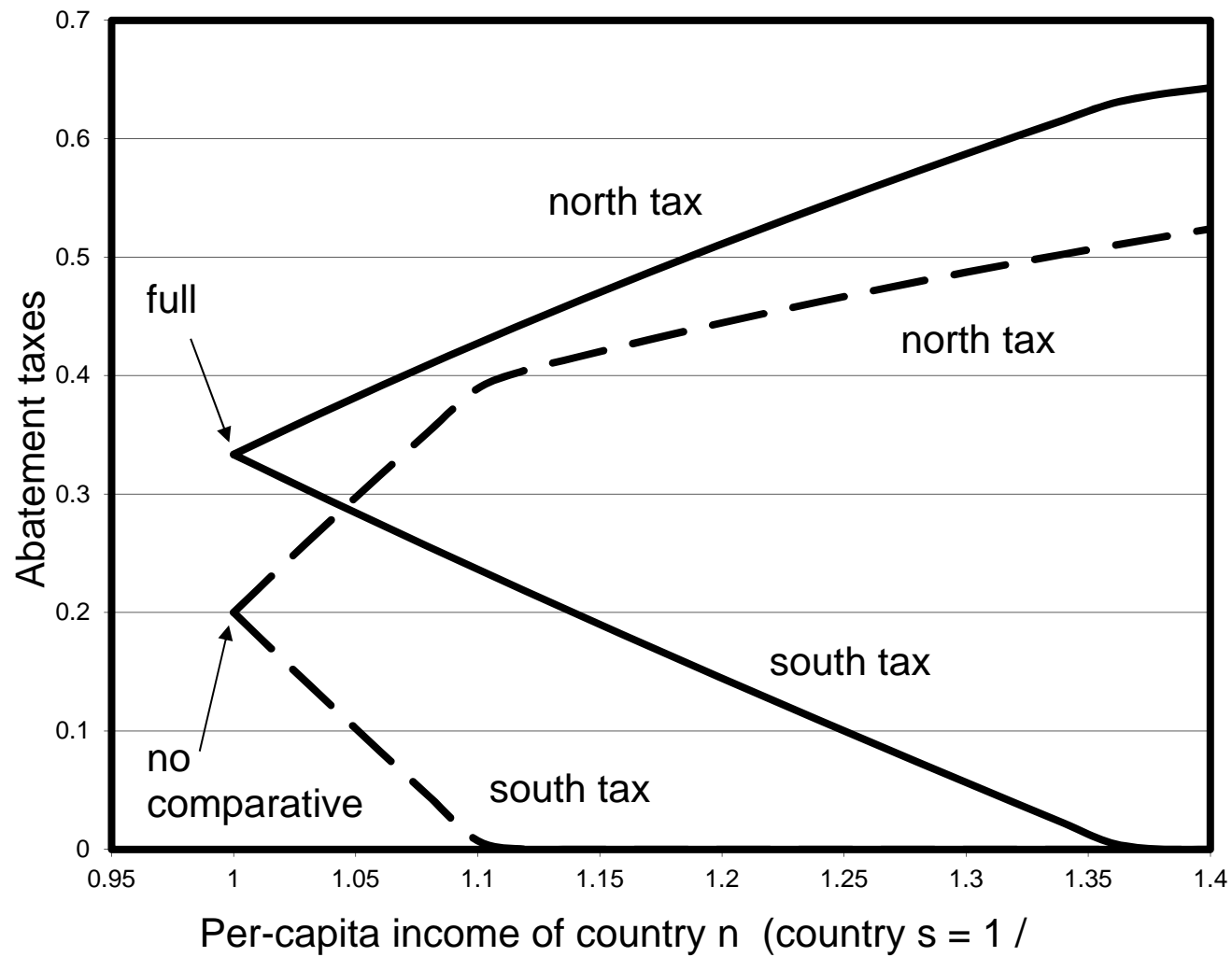


Figure 4: Welfare, environmental quality

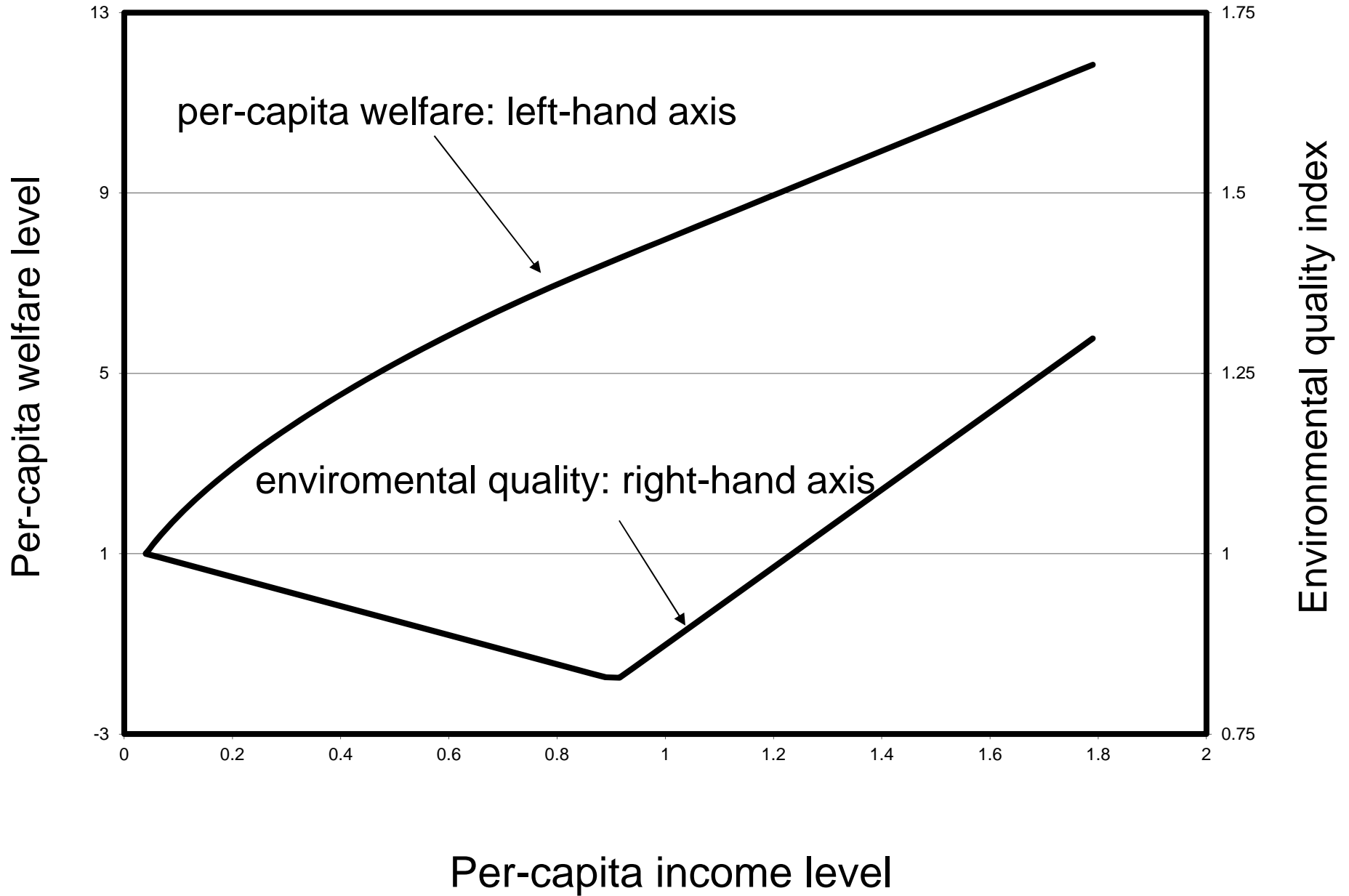


Figure 5: Policy Leakage, Border Tax Adjustments, Bilateral Protection

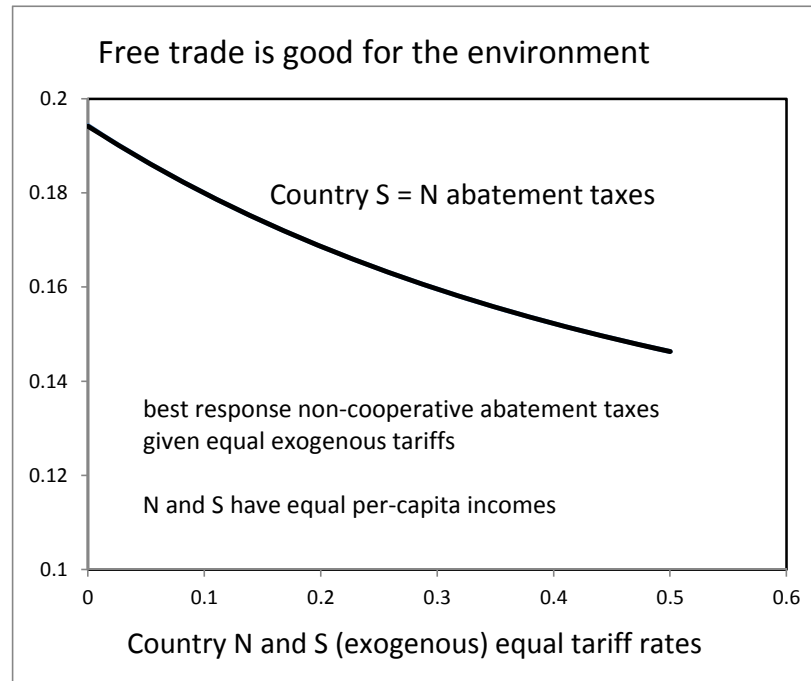
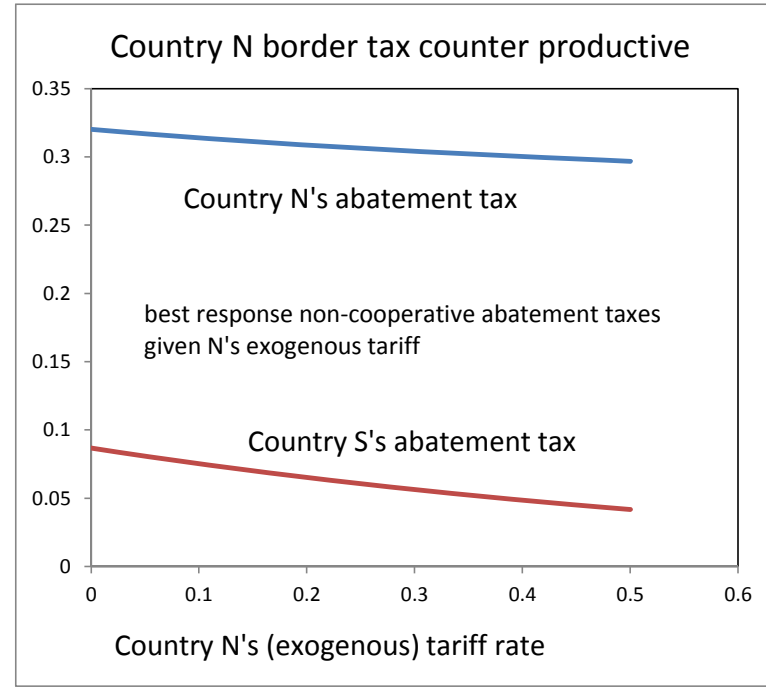
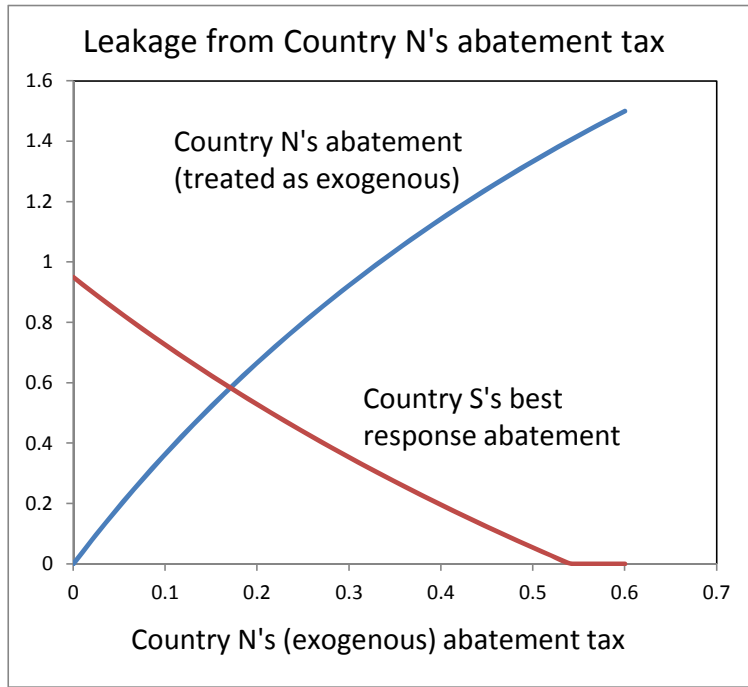


Figure 6: Effect of differing per capita income on equilibria

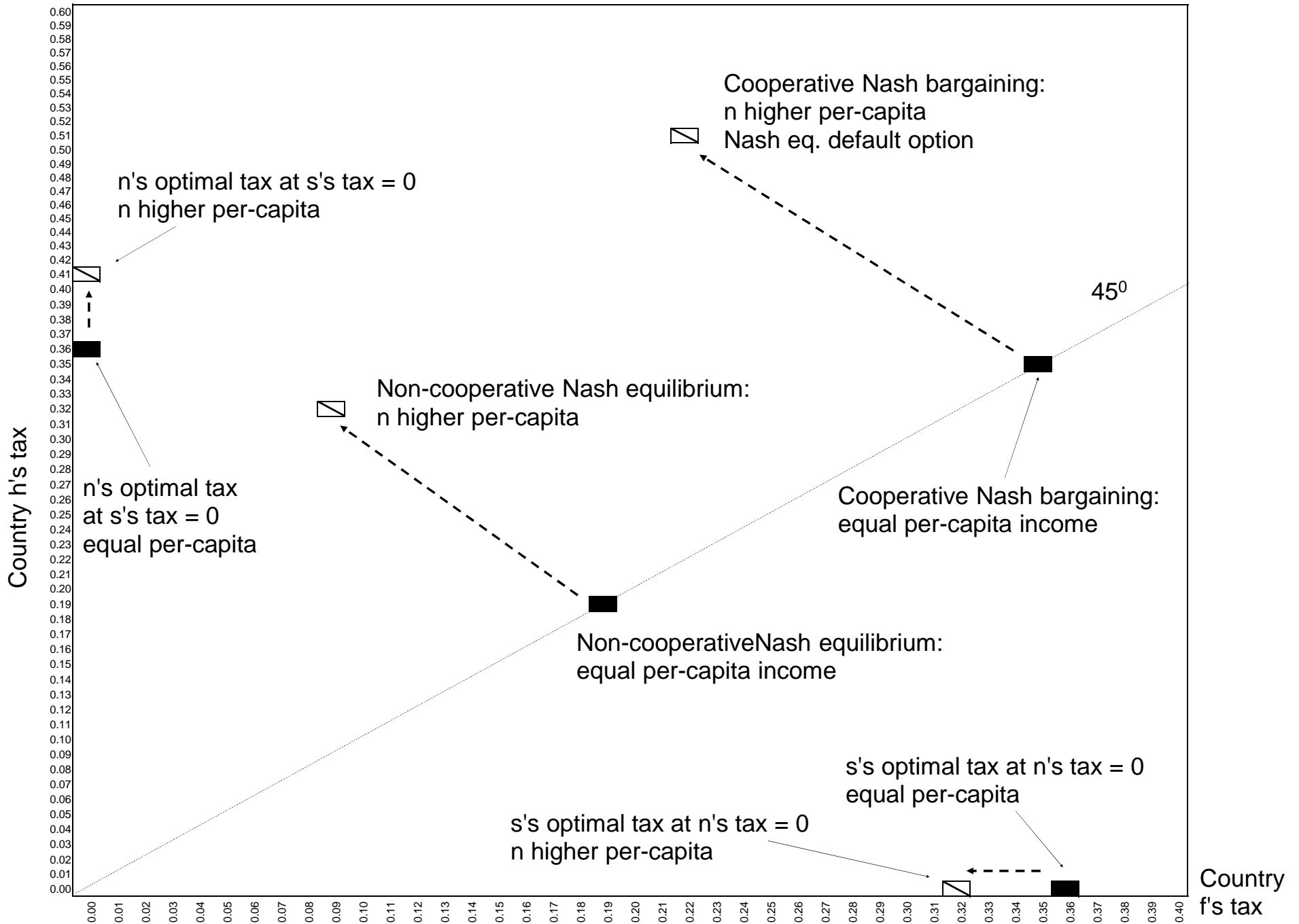


TABLE 1: Welfare and abatement taxes under alternative scenarios

	Base case: h's per-capita income 1.5 that of country f		Alternative case 1: no comparative advantage		Alternative case 2: h's per-capita income 10 times that of f	
	Welfare h	Welfare f	Welfare h	Welfare f	Welfare h	Welfare f
PTAX = 0	1.000	1.000	1.000	1.000	1.000	1.000
Non-cooperative NE	1.210	1.196	1.136	1.120	1.161	1.000
Coop Nash NE outside option	1.239	1.215	1.197	1.255	1.161	1.000
CD world welfare index max*	1.291	1.173	1.290	1.173	1.261	0.936
Rawlsian world welfare max**	1.225	1.225	1.237	1.228	1.161	1.000
TAX RATES	PTAXH	PTAXF	PTAXH	PTAXF	PTAXH	PTAXF
Non-cooperative NE	0.32	0.09	0.24	0.03	0.41	0
Coop Nash NE outside option	0.51	0.22	0.51	0.22	0.41	0
CD world welfare index max*	0.35	0.35	0.35	0.35	0.20	0.20
Rawlsian world welfare max**	0.57	0.20	0.48	0.28	0.41	0

*Cobb-Douglas index used is $(WELH)^{0.5} \cdot (WELF)^{0.5}$; welfare h,f normalized to 1 at zero taxes

**Rawlsian welfare index is $\text{MIN}(WELH, WELF)$; welfare h,f normalized to 1 at zero taxes

TABLE 2 Welfare and abatement taxes under alternative linking scenarios
 Country n has five times the total income and per-capita income of country s

	Welfare n	Welfare s	PTAXN	PTAXS	TARN	TARS
No intervention	0.731	0.788	0	0	0	0
Non-cooperative Nash each county coordinate: their tax and tariff	1.000	1.000	0.386	0	1.567	0.196
Cooperative Nash using non-cooperative outcome as disagreement outcome						
Isolated environment negotiation	1.001	1.008	0.451	0.036	1.567	0.196
Isolated tariff negotiation	1.008	1.048	0.386	0	1.015	0
Bargain over TARN and PTAXS	1.005	1.025	0.386	0.150	0.692	0.196
Bargain over all four instruments	1.009	1.059	0.452	0.034	1.015	0
Bargain over all four instruments with transfer s to n (= 22.0% of s's income)	1.024	1.075	0.541	0	0	0

\$TITLE:LINK-MPEC.GMS cooperation over all four policy instruments

\$ONTEXT INITIAL CALIBRATION (countries h = n in paper, f = s in paper)
calibrated to initial abatement taxes constrained to equal zero
Rows = markets; row sums zero: zero profits
Columns = sectors; column sums zero: market clearing, income balance

	ZH	ZF	XHF	XFF	YFF	YFH	WH	WF	CONSH	CONSF
PXH	180		-80				-100			
PYH	20					80	-100			
PXF		20	80					-100		
PYF		180				-80		-100		
PWH							300		-300	
PWF								300		-300
PLH	-200								200	
PLF		-200								200
PENVH							-100		100(a)	
PENVF								-100		100(b)

(a):100 = 100 units of initial E endowment plus e_0 = 100 equals 200
 minus 100 units of pollution generated by production = 100 for WH

(b):100 = 100 units of initial E endowment plus e_0 = 100 equals 200
 minus 100 units of pollution generated by production = 100 for WF

\$OFFTEXT

PARAMETERS

VALH per-capita income paramater for the S-G preferences - h
 VALF per-capita income paramater for the S-G preferences - f
 SIGMA elasticity of transformation between X and Y
 ENDOWLH factor endowment of country h
 ENDOWLF factor endowment of country f
 WELHN disagreement outcome for country h in bargaining
 WELFN disagreement outcome for country f in bargaining;

VALH = 1;
 VALF = 1;
 SIGMA = 1;
 ENDOWLH = 200; ENDOWLF = 200;
 WELHN = 1;
 WELFN = 1;

VARIABLES

WELH welfare of country h
 WELF welfare of country f
 WELG value of Nash bargaining objective function;

POSITIVE VARIABLES

ZH activity for composite X-Y commodity output in h
 ZF activity for composite X-Y commodity output in f
 XHF export activity: X from h to f
 YHF export activity: Y from h to f
 XFH export activity: X from f to h
 YFH export activity: Y from f to h
 WH welfare index for h (activity for "production" of WH)
 WF welfare index for h (activity for "production" of WF)
 ABH abatement activity for h
 ABF abatement activity for f

PZH price of Z (composite X-Y) in h
 PZFH price of Z (composite X-Y) in h
 PXH price of X in h
 PXFH price of X in f
 PYH price of Y in h
 PYFH price of Y in f
 PLH price of composite factor L in h
 PLFH price of composite factor L in f
 PWH price of welfare in h (real consumer price index)
 PWFH price of welfare in f (real consumer price index)

PABH price (cost) of the abatement activity in h
 PABFH price (cost) of the abatement activity in f
 PENVH consumer valuation of environmental quality in h
 PENVFH consumer valuation of environmental quality in h

CONSH consumer income in h
 CONSFH consumer income in f

POLRED pollution reduction due to combined abatement in h and f
 POL pollution generated by production of ZH and ZFH

PTAXH pollution tax in h - receipts allocated to abatement in h
 PTAXFH pollution tax in f - receipts allocated to abatement in f
 TARH tariff in country h
 TARFH tariff in country f;

EQUATIONS

WELFAREH welfare of country h
 WELFAREFH welfare of country f
 WELFAREG Nash bargaining objective function

COSTZH pricing equation for ZH (cost =G= price) - comp var ZH
 COSTZFH pricing equation for ZFH (cost =G= price) - comp var ZFH
 COSTXHFH pricing equation for XHFH (cost =G= price) - comp var XHFH
 COSTYHFH pricing equation for YHFH (cost =G= price) - comp var YHFH
 COSTXFHH pricing equation for XFHH (cost =G= price) - comp var XFHH
 COSTYFHH pricing equation for YFHH (cost =G= price) - comp var YFHH

COSTWH pricing equation for WH (cost =G= price) - comp var WH
 COSTWFH pricing equation for WH (cost =G= price) - comp var WH

COSTABH pricing equation for ABH (cost =G= price) - comp var ABH
 COSTABFH pricing equation for ABFH (cost =G= price) - comp var ABFH

MKTPZH market clearing equation for ZH - complementary variable PZH
 MKTPZFH market clearing equation for ZFH - complementary variable PZFH
 MKTPXH market clearing equation for XH - complementary variable PXH
 MKTPYH market clearing equation for YH - complementary variable PYH
 MKTPXFH market clearing equation for XF - complementary variable PXFH
 MKTPYFH market clearing equation for YF - complementary variable PYFH

MKTPLH market clearing equation for LH - complementary variable PLH
 MKTPLFH market clearing equation for LF - complementary variable PLFH
 MKTPWH market clearing equation for WH - complementary variable PWH
 MKTPWFH market clearing equation for WF - complementary variable PWFH

MKTPABH mkt clear for abatement in h: taxes = expenditure - comp PABH
 MKTPABF mkt clear for abatement in F: taxes = expenditure - comp PABF
 MKTENVH mkt clear environment in h: supply = demand - comp PENVH
 MKTENVF mkt clear environment in f: supply = demand - comp PENVF

ICONSH income balance equation in h - complementary variable ICONSH
 ICONSF income balance equation in f - complementary variable ICONSF

APOLRED pollution reduction = abatement by h and f - comp var POLRED
 APOL pollution generated by production by h and f - comp var POL;

**specify equations and inequalities*

WELFAREH.. WELH =E= WH;
 WELFAREF.. WELF =E= WF;
 WELFAREG.. WELG =E= ((WH - WELHN)*(WF - WELFN));

COSTZH.. 200*PLH =G= 200*PZH;
 COSTZF.. 200*PLF =G= 200*PZF;
 COSTXHF.. 80*PXH*(1+TARF) =G= 80*PXF;
 COSTYHF.. 80*PYH =G= (80*0.999)*PYF;
 COSTXFH.. 80*PXF =G= (80*0.999)*PXH;
 COSTYFH.. 80*PYF*(1+TARH) =G= 80*PYH;

COSTWH.. 300*((PXH*(1+PTAXH))**(1/3) * (PYH*(1+PTAXH))**(1/3)
 * PENVH**(1/3)) =G= 300*PWH;
 COSTWF.. 300*((PXF*(1+PTAXF))**(1/3) * (PYF*(1+PTAXF))**(1/3)
 * PENVF**(1/3)) =G= 300*PWF;

COSTABH.. 100*PLH =G= 100*PABH;
 COSTABF.. 100*PLF =G= 100*PABF;

MKTPZH.. 200*PZH =E= 200*(0.9*PXH**(1+SIGMA)
 + 0.1*PYH**(1+SIGMA))**(1/(1+SIGMA));
 MKTPZF.. 200*PZF =E= 200*(0.1*PXF**(1+SIGMA)
 + 0.9*PYF**(1+SIGMA))**(1/(1+SIGMA));
 MKTPXH.. 0.9*200*ZH*(PXH/PZH)**SIGMA - 80*XHF + 80*XFH
 =G= 100*WH*PWH/(PXH*(1+PTAXH));
 MKTPYH.. 0.1*200*ZH*(PYH/PZH)**SIGMA - 80*YHF + 80*YFH
 =G= 100*WH*PWH/(PYH*(1+PTAXH));
 MKTPXF.. 0.1*200*ZF*(PXF/PZF)**SIGMA - 80*XFH + 80*XHF
 =G= 100*WF*PWF/(PXF*(1+PTAXF));
 MKTPYF.. 0.9*200*ZF*(PYF/PZF)**SIGMA - 80*YFH + 80*YHF
 =G= 100*WF*PWF/(PYF*(1+PTAXF));

MKTPLH.. ENDOWLH =G= 200*ABH + 200*ZH;
 MKTPLF.. ENDOWLF =G= 200*ABF + 200*ZF;
 MKTPWH.. 300*WH =G= CONSH/PWH;
 MKTPWF.. 300*WF =G= CONSF/PWF;

MKTPABH.. 200*PABH*ABH =G= PTAXH*200*WH*PWH/(1+PTAXH);
 MKTPABF.. 200*PABF*ABF =G= PTAXF*200*WF*PWF/(1+PTAXF);
 MKTENVH.. (100+100*VALH) + 100*POLRED - 100*POL =G= 100*WH*(PWH/PENVH);
 MKTENVF.. (100+100*VALF) + 100*POLRED - 100*POL =G= 100*WF*(PWF/PENVF);

```

ICONSH..  CONSH =E=  ENDOWLH*PLH +((100+100*VALH)+100*POLRED-100*POL)*PENVH
           + 80*YFH*PYF*TARH;
ICONSF..  CONSF  =E=  ENDOWLF*PLF +((100+100*VALF)+100*POLRED-100*POL)*PENVF
           + 80*XHF*PXH*TARF;

APOLRED.. POLRED =G=  4*(ABH + ABF);
APOL..    POL    =G=  (ZH + ZF)/2;

```

**declare model definition, including matching equations and variables
 in the economics equilibrium constraint set (general-equilibrium model)

MODEL NHD /

```

WELFAREG
WELFAREH
WELFAREF
COSTZH.ZH
COSTZF.ZF
COSTXHF.XHF
COSTYHF.YHF
COSTXFH.XFH
COSTYFH.YFH
COSTWH.WH
COSTWF.WF
COSTABH.ABH
COSTABF.ABF

```

```

MKTPZH.PZH
MKTPZF.PZF
MKTPXH.PXH
MKTPYH.PYH
MKTPXF.PXF
MKTPYF.PYF
MKTPLH.PLH
MKTPLF.PLF
MKTPWH.PWH
MKTPWF.PWF

```

```

MKTPABH.PABH
MKTPABF.PABF
MKTENVH.PENVH
MKTENVF.PENVF

```

```

ICONSH.CONSH
ICONSF.CONSF

```

```

APOLRED.POLRED
APOL.POL /;

```

**set starting values*

```

ZH.L = 1;  ZF.L = 1;  XHF.L = 1;  YHF.L = 0;  XFH.L = 0;  YFH.L = 1;
WH.L = 1;  WF.L = 1;  ABH.L = 1;  ABF.L = 1;

```

```

PZH.L = 1;  PZF.L = 1;  PXH.L = 1;  PXF.L = 1;  PYH.L = 1;  PYF.L = 1;
PLH.L = 1;  PLF.L = 1;  PWH.L = 1;  PWF.L = 1;

```

```

PABH.L = 1;  PABF.L = 1;  PENVH.L = 1;  PENVF.L = 1;
CONSH.L = 300;  CONSF.L = 300;  POLRED.L = 0;  POL.L = 1;

```

**choose numeraire*

PXH.FX = 1;

VALH = 1;

VALF = 1;

ENDOWLH = 200;

ENDOWLF = 200;

** simplist scenario: outside options are zero*

** equivalenet to maximzing Cobb-Douglas world welfare function*

WELHN = 0;

WELFN = 0;

OPTION MPEC = NLPEC;

** benchmark replication with ptax constrained to = 0*

PTAXH.FX = 0;

PTAXF.FX = 0;

** solve command specifying maximand*

SOLVE NHD USING MPEC MAXIMIZING WELG;

** allow taxes to be set endogenously*

PTAXH.LO = 0; PTAXH.UP = **+INF**;

PTAXF.LO = 0; PTAXF.UP = **+INF**;

** solve for joint max*

SOLVE NHD USING MPEC MAXIMIZING WELG;