Location Equilibrium with Endogenous Rent Seeking

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Abstract

This paper analyzes the location of manufacturing activities when regional policy is determined by endogenous rent seeking. Once political factors such as lobbying for government transfers to regions are included in an economic geography framework with size asymmetries, the standard prediction that the larger region becomes the core when trade barriers are reduced does not hold anymore. The establishment of manufacturing production in the economically smaller region is increasing in the level of regional integration once trade becomes freer than a certain threshold value, and, when free trade prevails, the relocation of industry takes place up to the point where there are as many firms operating in the South as in the North. Furthermore, the introduction of lobbying for government subsidies slows down the agglomeration process, whereas the home market magnification effect (Baldwin, 2000) becomes weaker.

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1 Introduction

As is well known, the standard models of the New Economic Geography predict that in some circumstances smaller peripheral economies can be deindustrialized from economic integration. In contrast, the current paper argues that smaller regions are better able to capture the political gains through a rent-seeking contest for government transfers, which slows down or even reverses the agglomeration process implied by the conventional geography models. Moreover, this result accords well with the observation that in many European countries regions of low population density receive subsidies that are not justified by their size, which prevents and delays the clustering of economic activity.

Interest groups play a prominent political role in all democratic societies. Their activities are many and varied; they contribute to campaigns; they lobby politicians and they educate the public about issues and candidates. By these means and more, the groups seek to influence the political system in ways that further the interests of their members. According to all indications, the participation of interest groups in the policy-making process has been growing worldwide. The number of organizations that employ representatives in Washington, Brussels, and other capital cities has increased significantly in recent years. So too has the number of registered lobbyists, as has the total amount of spending on lobbying. For example, the number of organizations that have each spent at least USD 2 million to lobby the US government increased from 117 in 1998 to 131 in 2000. Moreover, businesses, trade organizations, labor unions and other interest groups reported spending USD 1.35 billion on lobbying political decision makers in Washington during 1998, compared to 1.55 billion throughout 2000 (The Center for Responsive Politics, 2000).[1] Further analysis by the Center for Responsive Politics shows that matters dealing with taxation, followed by budget and appropriation concerns, are the most heavily lobbied issues (The Center for Responsive Politics, 2002). Hence, vast resources are being spent on pressure for a specific state tax and regional policy that favor individual businesses, selected industries, or certain geographic regions. Regions are consequently one of many stakeholders seeking to influence the government and corporate decision makers on issues such as business expansion and relocation decisions.

[1] The data is available at http://www.opensecrets.org/pubs/lobby00/index.asp (accessed March 25, 2004).

At the same time, many countries devote a large part of their national budget to regional policy. These policies are designed to support the development of backward regions that are substantially below the average of other regions. In addition to the national redistribution scheme among European countries, a significant share of the EU budget is also allocated to reduce regional imbalances and promote regional development. For example, 33 billion euros or 34 percent of the EU budget for 2001 was set aside to improve the economic conditions in areas facing structural difficulties. A considerable fraction of the budget devoted to regional policy at the national level, as well as at the EU level, consists of subsidies to private firms located in poor regions. The main purpose of these transfers is to attract firms, based on the assumption that this will reduce the regional inequalities from an increased economic integration.

Although large sums apparently are being spent on support to regions with poor economic conditions, almost 50 percent of these funds in 1998 were directed towards member states with a per capita GDP at or above the EU average (Siebert, 2001). Moreover, according to Homburg (1997), GNP per capita had, on the one hand, no effect on net transfers within the EU in 1992, while, on the other hand, population size was significantly negatively related to the distribution of regional benefits. An explanation put forward by Robert-Nicoud and Sbergami (2004) emphasizes that the population in smaller regions is more homogenous politically. Politicians will therefore find that a given subsidy level can buy more votes when the subsidies are distributed to firms based in these areas, where there are relatively more swing voters. Cadot et al. (2001) provide empirical support for this theory. Using French data, they show that electoral concerns as well as rent-seeking expenditures are significant determinants of the spatial allocation of regional infrastructure investments. They, however, take the location of industry as given, but, nevertheless, bring focus to a relationship between influence activities, regional policy and economic geography.

In the present paper this issue is addressed by analyzing the interaction between the location of economic activity and regional integration, when regional policy is determined by endogenous rent seeking. The economic model, with monopolistic competition and asymmetric-sized regions, where capital is inter-regionally mobile, gives rise to the home market effect (Krugman, 1980). That is, as trade barriers gradually are lowered, the allocation of industry shifts towards the larger region (North). Further liberalization continues to benefit the larger region until all industry agglomerates in the

North. However, when lobbying for government transfers is allowed for in the model, this standard prediction does not hold anymore. The equilibrium location of industry in the economically smaller region (South) is, in fact, increasing in the level of economic integration once trade becomes freer than a certain critical value. Furthermore, lobbying for regional subsidies slows down the agglomeration process, whereas the home market magnification effect becomes weaker. The explanation for these results is that the larger size of a region reflects a political weakness; rent seeking agents from a larger region are less willing to expend resources for a marginal increment of subsidies to its industrial sector because of a higher rate of rent dissipation in the political equilibrium. This will shift the equilibrium policy variable in favor of the South, and, as trade barriers are reduced, government transfers to manufacturing production become relatively more efficient in attracting industrial activity. When free trade prevails, the relocation of manufacturing activities takes place up to the point where there are as many firms operating in the South as in the North.

The rest of this paper is organized as follows. The next section presents the economic geography model and solves for the equilibrium location when the regional policy variable is taken as exogenous. Section 3 sets out the political game that endogenously decides the size and direction of the policy instrument. In Section 4, the political model is integrated into the economic model and the two are solved together. Section 5 concludes the paper.

2 The Economic Model

The basic economic model uses the Martin and Rogers (1995) framework, which is based on the Flam-Helpman (1987) version of the Dixit-Stiglitz (1977) model of monopolistic competition.

2.1 Assumptions

There are two regions, two sectors and two factors. Specifically the two regions, North (N) and South (S), belong to the same country (or federation of countries) and are endowed with two factors, labor (L) and capital (K). The regions are symmetric in terms of tastes, technology, openness to trade, but differ in their factor endowments; the North is a scaled-up version of the South. That is, the regions may be of different size but they have identical capital-labor ratios. In particular, the North's endowment of both capital and labor is $\lambda > 1$ times the South's endowment. For this reason λ can be interpreted as the relative economic strength of the North.

The two sectors are referred to as agriculture and manufacturing (or industry). The agricultural sector is assumed to produce a homogenous good under constant returns to scale and perfect competition using a_A units of labor per unit of output. Labor is the only input and this good is chosen as a numeraire. The manufacturing sector uses both labor and capital to produce a differentiated good under increasing returns to scale and monopolistic competition. Following Flam and Helpman (1987), production of each differentiated good involves a one-time fixed cost consisting of one unit of K and a per-unit-of-output cost of a_M units of L. The implied cost function of each industrial firm is therefore given by:

$$\pi + wa_M x, \tag{2.1}$$

where π and w are the reward to capital and labor, and x is the firm level output.

Physical capital can move between regions but capital owners are immobile. Thus, when pressures arise to concentrate production in one region, capital will move, but all of its reward will be repatriated to its region of origin. Labor, on the other hand, can move freely between sectors but is immobile between regions. Total supply of capital and labor in the economy is fixed, with the nation's endowments denoted by K_W and L_W. Because each industrial variety requires one unit of capital, the share of the nation's capital stock employed in a single region exactly equals the region's share of the whole (national) manufacturing sector. Consequently, the North's share of industry can be used, i.e., $s_N \equiv \frac{n_N}{n_N+n_S}$, where n_N and n_S denote the number of industrial firms in the North and the South respectively, to represent the share of capital employed in the North and the share of all varieties made in the North.

Output in the agricultural sector is traded at no cost, while inter-regional trade in the differentiated output is subject to an iceberg transportation cost. Hence, in order to sell one unit of the differentiated good in the other region, $\tau > 1$ units need to be shipped.

The representative consumer in each region has preferences according to:

$$U = C_M^{\mu} C_A^{1-\mu}, \tag{2.2}$$

where $\mu \in (0,1)$ and C_A is consumption of the homogenous good. Consumption of manufactures enters the utility function through the index C_M, which is defined by:

$$C_M \equiv \left(\sum_{i=1}^{n_W} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2.3}$$

where $n_W = n_N + n_S$ denotes the large number of industrial varieties consumed, fixed by the nation's total supply of capital, c_i is the amount of variety i consumed, and $\sigma > 1$ being the constant elasticity of substitution between any two varieties.

Now assuming that a government can subsidize manufacturing production in both regions at the same time, these subsidies are proportional to the reward to capital and distributed on a per-firm basis independent of output. Since the one-time fixed cost consists of one unit of K, such subsidies actually represent a subsidy to capital. Let π_N (π_S) denote the before-subsidy reward to capital when producing in the North (South) and let $\theta \equiv \frac{\pi_N}{\pi_S} = \frac{1+z_S}{1+z_N}$, where $\theta \in \Re_{++}$ and z_j is the units of per-firm subsidies proportional to the reward to capital, awarded to a typical firm located in region $j = N, S$. Subsidies defined in this way thus entitle the firm to z_j units of subsidies per unit of π_j. Hence, the total value of government benefits to a firm can either increase when the reward to capital rises, or because each firm receives more units of subsidies. When production in the South is net subsidized, the condition $\theta > 1$ must hold, and so $\theta - 1$ is the ad valorem net subsidy given to capital owners who produce in that region, regardless of their origin. When production in the North is net subsidized, the condition $0 < \theta < 1$ has to be fulfilled, and accordingly $\frac{1}{\theta-1}$ is the ad valorem net subsidy given to capital owners who produce in that region.

The subsidies in this model are paid for by lump-sum taxation, and since each unit of labor is identified with an individual, the per-capita tax can be expressed as a per-unit-of-labor tax. Moreover, the government's budget is always balanced, so the level of taxation depends on the number of subsidies transferred. By assumption, there are no savings in the model and therefore expenditures equal disposable income. Consequently, expenditures in region j are given by:

$$E_j = w_j L_j + \pi_j (1 + z_j) K_j - T L_j, \tag{2.4}$$

where T is the countrywide lump-sum tax paid by the representative consumer.

2.2 The Economic Equilibrium

The unit factor requirement of the homogenous good is one unit of labor ($a_A = 1$). This good is freely traded and since it is also chosen as a numeraire, $p_A = w = 1$ in both regions.

Each consumer spends a share μ of his income on manufactures, and the total demand for variety i of the differentiated good is therefore:

$$c_i = \frac{p_i^{-\sigma} \mu E_j}{\sum_{k=1}^{n_W} p_k^{1-\sigma}}, \tag{2.5}$$

where p_i is the price of variety i. Profit maximization yields:

$$p = \frac{\sigma}{\sigma - 1} w a_M, \tag{2.6}$$

and

$$p^* = \frac{\sigma}{\sigma - 1} \tau w a_M, \tag{2.7}$$

for each differentiated commodity sold in the home and export market respectively. Choosing $a_M = \frac{\sigma-1}{\sigma}$, and using $w = 1$ give the pricing rules for firms in the manufacturing sector: $p = 1$ and $p^* = \tau$.

Since physical capital is used only in the fixed cost component of industrial production, the reward to capital is the Ricardian surplus of a typical variety, i.e., the operating profit of a variety. With a fixed capital stock and free entry the reward to capital will be bid up to the point where the entire operating profit goes to capital. Under Dixit-Stiglitz competition this operating profit is defined as the value of sales divided by σ. That is $\pi_j = \frac{x_j}{\sigma}$, where x_j is the scale of production of a representative industrial firm in region j.

By assumption, the ownership of capital is diversified nationally. Therefore with no savings, disposable income or expenditures in region j can be written as:

$$E_j = s_{E_j}[L_W + \bar{\pi}(1 + z_W)K_W - TL_W], \tag{2.8}$$

where $s_{E_j} = \frac{E_j}{E_W}$ and $\bar{\pi}(1 + z_W)$ is the national subsidy-included return to capital, which is determined by the condition $\bar{\pi}(1 + z_W)K_W = \frac{\mu E_W}{\sigma} + TL_W$.

So, with free entry in the industrial sector, the national subsidy-included return to capital is a function of the sum of the national operating profit and the total amount of subsidies transferred to all the manufacturing firms in the economy (since the government's budget is balanced). Total expenditures in turn equal total disposable income: $E_W = L_W + \frac{\mu E_W}{\sigma}$, which implies that $E_W = \frac{\sigma L_W}{\sigma - \mu}$. The national subsidy-included return to capital is therefore:

$$\bar{\pi}(1 + z_W) = \frac{\mu L_W}{(\sigma - \mu) K_W} + \frac{T L_W}{K_W}. \tag{2.9}$$

Using (2.9) in (2.8) solves for the expenditures in region j:

$$E_j = s_{E_j} \left(\frac{\sigma}{\sigma - \mu} \right) L_W. \tag{2.10}$$

Using the demand function and the optimal prices give the reward to capital or the operating profit in equilibrium:

$$\pi_N = \frac{x_N}{\sigma} = \frac{1}{\sigma} \left(\frac{\mu E_N}{n_N + \phi n_S} + \frac{\phi \mu E_S}{\phi n_N + n_S} \right), \tag{2.11}$$

and

$$\pi_S = \frac{x_S}{\sigma} = \frac{1}{\sigma} \left(\frac{\phi \mu E_N}{n_N + \phi n_S} + \frac{\mu E_S}{\phi n_N + n_S} \right), \tag{2.12}$$

$\tau^{1-\sigma} \equiv \phi \in [0, 1]$ is a measure of the freeness of inter-regional trade, where 0 corresponds to infinite trade barriers and 1 represents free trade. Substituting (2.10) into (2.11) and (2.12) to obtain:

$$\pi_N = a L_W \left[\frac{s_{E_N}}{s_N + \phi(1 - s_N)} + \frac{\phi(1 - s_{E_N})}{\phi s_N + (1 - s_N)} \right], \tag{2.13}$$

and

$$\pi_S = a L_W \left[\frac{\phi s_{E_N}}{s_N + \phi(1 - s_N)} + \frac{(1 - s_{E_N})}{\phi s_N + (1 - s_N)} \right], \tag{2.14}$$

where $a \equiv \frac{\mu}{\sigma - \mu}$, s_{E_N} and $1 - s_{E_N} = s_{E_S}$ represent the North and the South's share of national expenditures, s_N and $1 - s_N = s_S$ being the North and the South's share of the national manufacturing sector. With one unit of capital per variety, s_j is also defined as the share of the national capital stock employed in region j.

With perfect capital mobility and when manufacturing production takes place in both regions, the location condition requires that capital employed in the North must earn the same subsidy-included rate of return as the capital in the South: $\pi_N(1 + z_N) = \pi_S(1 + z_S)$. The distribution of industry that solves this condition is:[2]

$$s_N = \frac{s_{E_N}(1-\phi^2) - \phi(\theta - \phi)}{(1-\phi)[(\theta - \phi) - s_{E_N}(\theta - 1)(1+\phi)]}. \tag{2.15}$$

Let $\theta = 1$ (i.e., no net subsidies transferred) and differentiate (2.15) with respect to s_{E_N}:

$$\frac{\partial s_N}{\partial s_{E_N}}|_{\theta=1} = \frac{1+\phi}{1-\phi} > 1, \tag{2.16}$$

which demonstrates the home market effect. Thus s_N increases more than proportionate to s_{E_N} for $\phi \in (0, 1)$, and this effect becomes stronger as trade barriers are reduced (home market magnification). This means that, even if one region is just slightly larger than the other, it will obtain the entire manufacturing industry if transaction costs are low enough.

Now set $s_{E_N} = 1/2$ (that is, imposing equal regional size) and differentiate (2.15) with respect to θ:

$$\frac{\partial s_N}{\partial \theta}|_{s_{E_N}=\frac{1}{2}} = -\frac{(1+\phi)^2}{(1+\theta)^2(\phi - 1)^2} < 0. \tag{2.17}$$

(2.17) illustrates the effect of subsidies on the geographical equilibrium and implies that the location of manufacturing activities in the North is decreasing in net subsidies distributed to the South, when both regions are symmetric in terms of their factor endowments. By differentiating (2.17) with respect to ϕ it can be shown that trade liberalization magnifies this effect as well. Consequently, assuming equal regional size, the freer the trade, the smaller the net subsidy required to attract all firms to the region favored by the government. However, it is obvious from (2.16) that a redistributive regional policy is necessary to offset the tendency of the smaller region to lose firms as the transaction costs fall because of the home market effect.

With the economic model specified it is now time to introduce the political game that aims to determine the size and direction of the net subsidy (θ).

[2] The solution to the location condition is similar to the one obtained in Baldwin et al. (2003), p. 453.

3 The Political Model

So far the regional policy variable (θ) has been taken as exogenous in the theoretical setting. It shall now be determined by a specific political process. To analyze the lobbying game a modification of the model by Wärneryd (1998) is used. The government distributes $TL_W = \Gamma$ units of subsidies (a rent) to the regions, giving the South a share α and the North a share $1-\alpha$. Thereupon individual or firm specific shares of the rent are determined through an internal regional contest. Hence, both regions attempt to divert the rent and once a particular region is awarded a share, each individual firm within the region will compete to obtain as high a proportion of it as possible. The rent thus clearly exhibits characteristics of both the public and private good; it is a public good in the first stage of the game, and it is a private good in the second. Aggregate group spending in the first stage determines the proportion of the rent allocated to a specific region. This is a public good to members of the region. In the second stage, individual rent seeking determines the proportion allocated to each firm.

Since the regional policy instrument under analysis is independent of output, and therefore represent a subsidy to capital or to setting up an industrial firm in one of the regions, only firms in the manufacturing sector will engage in rent-seeking activities.

3.1 Assumptions

Consider now the specific assumptions of the 2-stage game described above. At the first stage the government sets the allocation α. It does so in response to aggregate rent-seeking expenditures made by region j. Spending (bribes or maybe campaign contributions) may take the form of cash transfers, gifts or information to the government about policy matters of importance. Following Wärneryd (1998), let the first-stage expenditures by the typical firm i in region j be t_{ji}, and let $\mathbf{t}$ be the vector of all per-firm expenditures. The government then sets α so that:

$$\alpha(\mathbf{t}) = \begin{cases} \left(\sum_i t_{Si}\right) / \left(\sum_i t_{Ni} + \sum_i t_{Si}\right), & \text{if } \left(\sum_i t_{Ni} + \sum_i t_{Si}\right) > 0; \\ 1/2, & \text{otherwise.} \end{cases} \tag{3.1}$$

That is, region j obtains a share of the total number of subsidies equal to the ratio of the sum of the expenditures of its individual firms to the total first-stage expenditures. At the second and final stage, the share of each individual firm is determined through a local contest.

A subgame perfect equilibrium outcome of this game is solved by backward induction. At the final stage, α is given and any previous rent seeking expenditures are sunk. As a result, the firms in region j must play the unique symmetric equilibrium given their share of the total amount of subsidies. Using the contest model originally proposed by Tullock (1980), the expected share of firm i in region j is then given by:

$$\beta_{ji}(\mathbf{q}_j) = \begin{cases} q_{ji} / \left(\sum_{l=1}^{n_j} q_{jl}\right), & \text{if } \left(\sum_{l=1}^{n_j} q_{jl}\right) > 0; \\ 1/n_j, & \text{otherwise,} \end{cases} \tag{3.2}$$

where $\mathbf{q}_j = (q_{j1}, q_{j2}, ..., q_{jn_j}) \in \Re_+^{n_j}$ is the vector of individual investments made by firms in region j.

3.2 The Rent-Seeking Equilibrium

The risk neutral firm i in region S wishes to maximize the expected return from rent seeking according to:

$$Er_{Si}(\mathbf{q}_S) = \beta_{Si}(\mathbf{q}_S)\alpha\Gamma - q_{Si}, \tag{3.3}$$

for which the first order condition is:

$$\frac{\sum_{l \neq i}^{n_S} q_{Sl}\alpha\Gamma}{\left(\sum_{l=1}^{n_S} q_{Sl}\right)^2} = 1. \tag{3.4}$$

Let q_S be the common expenditure level of firms in the South in equilibrium:

$$q_S = \frac{(n_S - 1)\,\alpha\Gamma}{n_S^2}. \tag{3.5}$$

The equilibrium expected return of each firm in the South is then:

$$Er_S = \frac{\alpha\Gamma}{n_S^2}, \tag{3.6}$$

and for each firm in the North:

$$Er_N = \frac{(1-\alpha)\,\Gamma}{n_N^2}. \tag{3.7}$$

Hence, the expected return of entering the second stage is lower for a firm that produces in the larger region. The reason is that the subsidies are diluted by the number of firms in a region and because the larger region is more wasteful in the second stage. Consequently, there are fewer units available for the representative firm in the North, where each unit distributed is also worth less, since a relatively larger share of the transfers per firm will be dissipated in the second-stage equilibrium.

At the first stage, firms must take the expected outcome in the second stage into account when deciding on how much to expend in the aggregate. From the perspective of the first stage, the expected return of firm i in the South is thus:

$$E\tilde{r}_{Si}(\mathbf{t}) = \frac{\alpha(\mathbf{t})\Gamma}{n_S^2} - t_{Si}. \tag{3.8}$$

The optimal first-stage expenditure of a typical firm in the South is then given by the first-order condition:

$$\frac{\Gamma \sum_i t_{Ni}}{n_S^2 \left(\sum_i t_{Ni} + \sum_i t_{Si}\right)^2} = 1. \tag{3.9}$$

The corresponding condition for a representative firm in the North is:

$$\frac{\Gamma \sum_i t_{Si}}{n_N^2 \left(\sum_i t_{Ni} + \sum_i t_{Si}\right)^2} = 1. \tag{3.10}$$

Following the assumption that all rent-seeking firms in region j make the same equilibrium expenditures, let t_N denote the common expenditure level in the North and t_S in the South. Equilibrium then requires that:

$$\frac{\Gamma n_N t_N}{n_S^2 (n_N t_N + n_S t_S)^2} = 1, \tag{3.11}$$

and

$$\frac{\Gamma n_S t_S}{n_N^2 (n_N t_N + n_S t_S)^2} = 1. \tag{3.12}$$

The common first-stage rent-seeking expenditures in equilibrium made by firms in region S are then given by:

$$t_S = \frac{\Gamma n_N^2}{n_S (n_N^2 + n_S^2)^2}, \tag{3.13}$$

and for firms in the North:

$$t_N = \frac{\Gamma n_S^2}{n_N(n_N^2 + n_S^2)^2}. \tag{3.14}$$

The equilibrium allocation set by the government is accordingly:

$$\alpha = \frac{n_N^2}{n_N^2 + n_S^2}, \tag{3.15}$$

i.e., the South obtains a larger share of the subsidies, since there are relatively more industrial firms (or capital) operating in the North. The intuition for this result is that a dollar increment of the share at the first stage is worth more to the fewer firms located in the South, because less of it will be dissipated in the second-stage regional contest, than would be the case if it ended up in the larger region. Therefore, firms in the South are willing to expend relatively more resources to acquire such an increment.

The number of subsidies awarded to a typical firm in the South and the North respectively is then:

$$z_S = \frac{\Gamma n_N^2}{n_S\,(n_N^2 + n_S^2)}, \tag{3.16}$$

and

$$z_N = \frac{\Gamma n_S^2}{n_N\,(n_N^2 + n_S^2)}. \tag{3.17}$$

That is, government transfers per firm to region j are negatively related to capital employed in j, or the size of the manufacturing sector. The reason is that the expected return of entering the second stage in the lobbying game is lower for firms from a larger region because of the relatively higher rate of rent dissipation.

Using (3.16) and (3.17) to solve for the net transfers in equilibrium:

$$\theta_{NE} = \frac{\Gamma n_N^3 + n_N^3 n_S + n_N n_S^3}{\Gamma n_S^3 + n_N^3 n_S + n_N n_S^3}. \tag{3.18}$$

Differentiating (3.18) with respect to n_N and n_S gives:

$$\frac{\partial \theta_{NE}}{\partial n_N} = \frac{\Gamma n_S[3n_N(\Gamma + n_S) + n_S^3 + 2n_N]}{(\Gamma n_S + n_N n_S + n_N)^2} > 0, \tag{3.19}$$

and

$$\frac{\partial \theta_{NE}}{\partial n_S} = -\frac{\Gamma n_N [n_S^2(3\Gamma + 2n_S) + n_N^3 + 3n_N n_S^2]}{n_S^2 (\Gamma n_S^2 + n_N n_S^2 + n_N^3)^2} < 0. \tag{3.20}$$

Increasing the relative economic strength of region j thus reduces net transfers to j in equilibrium, and so regional factor abundance seems to give rise to a political weakness from a rent-seeking standpoint. The ratios $\frac{n_N}{n_S}$ and $\frac{n_S}{n_N}$ can therefore be interpreted as the relative political strength of the South and the North respectively.

This defines the rent-seeking equilibrium, which now makes it possible to solve for the model as a whole, i.e., for the distribution of industry implied by the political equilibrium (s_N^*).

4 The Political Economy Equilibrium

Next, the following 2-stage game is to be considered. At the first stage, firms relocate between regions in response to the level of economic integration (ϕ) and the relative shares of total expenditures (s_{E_N}), i.e., the relative economic strength of the North. At the second and final stage, the political process specified in Section 3 determines the number of subsidies (z_j) transferred to a representative firm in region j, and in the political equilibrium the allocation of net subsidies (θ_{NE}) is set according to (3.18). This structure is illustrated in Figure 1.

Figure 1: Timing of the game

Once again, a subgame perfect equilibrium outcome is solved by backward induction. At the final stage, the distribution of industry and accordingly n_N as well as n_S (the number of industrial firms in the North and the South respectively) are already determined by the first-stage relative economic strength of the North and the freeness of inter-regional trade. From the perspective of the second stage and by (3.18), the government must therefore take the first-stage equilibrium into account when net subsidies are distributed. The home market effect implies that the equilibrium share of industry in the North is increasing more than proportionate to its relative economic strength, and the bias is magnified as the transaction costs are lowered. This will, however, reduce the relative political strength of the North and increase net transfers to the South at the second stage by (3.18), which, to a certain extent, counterbalance the market-access advantage of producing in the larger region since $\frac{\partial s_N}{\partial \theta} < 0$.[3]

From the perspective of the first stage, the firms in region j must take the second-stage equilibrium into account when relocation decisions are made. Hence, the first-stage equilibrium is also affected by the relative political strength of region j at the second stage, which determines the size and direction of government net transfers. To solve for the distribution of industry implied by the rent-seeking equilibrium, $n_N^3 n_S + n_N n_S^3 \equiv \rho > 0$ is defined and (3.18) in (2.15) is used:

$$s_N^* = \frac{s_{E_N}\left(1-\phi^2\right) - \phi\left[\left(\frac{\Gamma n_N^{*\,3}+\rho}{\Gamma n_S^{*\,3}+\rho}\right) - \phi\right]}{(1-\phi)\left[\left[\left(\frac{\Gamma n_N^{*\,3}+\rho}{\Gamma n_S^{*\,3}+\rho}\right) - \phi\right] - s_{E_N}\left[\left(\frac{\Gamma n_N^{*\,3}+\rho}{\Gamma n_S^{*\,3}+\rho}\right) - 1\right](1+\phi)\right]}, \tag{4.1}$$

where the number of industrial firms at the second stage in the North and the South depends on the relocation decisions made at the first stage, and thus implicitly on the relative economic strength of the North and the level of inter-regional trade barriers exogenously determined in the first stage: $n_N^* = f(s_{E_N},\, \phi)$ and $n_S^* = f(s_{E_N},\, \phi)$. Taking the total derivative of (4.1) with respect to s_{E_N}, using $\frac{\partial s_N^*}{\partial s_{E_N}} \equiv \frac{\partial n_N^*}{n_W \partial s_{E_N}} \equiv -\frac{\partial n_S^*}{n_W \partial s_{E_N}}$ and rearranging terms, it follows that (proof in the Appendix):

Proposition 1: *In the political economy equilibrium the introduction of lobbying for government transfers slows down the agglomeration process:* $\frac{\partial s_N^*}{\partial s_{E_N}} < \frac{\partial s_N}{\partial s_{E_N}}$*, provided that* $\frac{\partial s_N}{\partial s_{E_N}} > 0$.

So, once rent-seeking activities are taken into consideration, the relative economic strength of a region obviously becomes a less effective determinant

[3] Differentiating (2.15) with respect to θ yields: $\frac{\partial s_N}{\partial \theta} = \frac{s_{E_N}\left(-1+s_{E_N}\right)(1+\phi)^2}{\left[\phi-\theta_{NE}+s_{E_N}(-1+\theta_{NE})(1+\phi)\right]^2} < 0$, for $s_{E_N} \in (0,1)$.

of industry location at any point on the integration path. By totally differentiating $\frac{\partial s_N^*}{\partial s_{E_N}}$ with respect to ϕ, it can also be established that (proof in the Appendix):

Proposition 2: *When regional policy is determined by endogenous rent seeking and the number of subsidies awarded to a typical firm in both regions is greater than or equal to one ($z_j \geq 1$), freer trade magnifies the degree of relocation that comes from a given shift in economic activity at a slower rate compared to the location equilibrium without lobbying for government subsidies. That is to say, the home market magnification effect is weaker allowing for endogenous rent seeking:* $\frac{\partial^2 s_N^*}{\partial\phi\partial s_{E_N}} < \frac{\partial^2 s_N}{\partial\phi\partial s_{E_N}}$*, given that* $\frac{\partial^2 s_N}{\partial\phi\partial s_{E_N}} > 0$, $\frac{\partial s_N}{\partial s_{E_N}} > 0$, $\frac{\partial^2 s_N}{\partial\phi\partial\theta_{NE}} < 0$ *and* $\frac{\partial s_N^*}{\partial\phi} > 0$.

The reason for these two results can be explained as follows. On the one hand, the equilibrium location of manufacturing production in region j is increasing in its first-stage share of national expenditures. On the other hand, this sole agglomeration force in the model tends to weaken the bargaining position in the rent-seeking contest at the second stage of the political economy game. To see this, note that θ_{NE} is increasing in n_N^* and decreasing in n_S^* by (3.19) and (3.20), while s_N^* decreases in θ_{NE} (see footnote 3). As a consequence, the South's relative political strength, determined in the second stage of the game, decreases the share of firms located in the North at the first stage, i.e., $\frac{\partial s_N^*}{\partial\left(n_N^*/n_S^*\right)} < 0$. Now, in the rent-seeking equilibrium the representative firm in the South is awarded more subsidies than the typical firm in the North by (3.16) and (3.17), and therefore the manufacturing sector in the South receives positive net transfers by (3.18). This increases the subsidy-included relative return to capital in the South by the location condition, which implies that the tendency of the smaller region to lose firms at the first stage, as the transaction costs fall, is offset by the fact that in the political equilibrium there is a lower rate of rent dissipation in the South. Nevertheless, in the political economy equilibrium the larger region will always end up attracting relatively more firms as long as its economic strength more than compensates for its political weakness, and this bias is magnified when trade barriers are reduced by the home market magnification effect. But, as stated in Proposition 2, the location of industry will, in this case, respond less to lower transaction costs compared to the outcome without lobbying for government transfers, because the effect of reduced trade barriers is now partly offset by a higher rate of rent dissipation in the larger region.

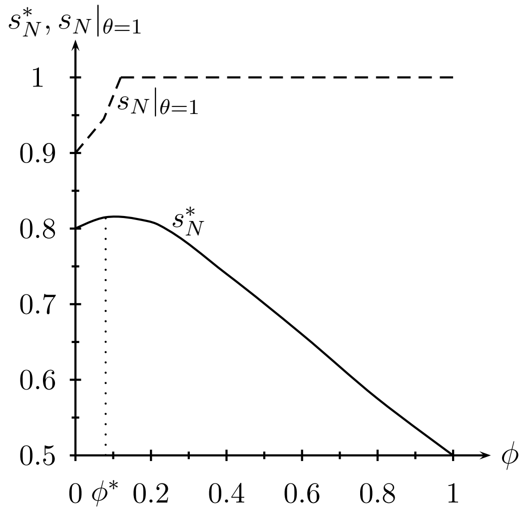
Whatever the relative strength of the two regions, whether economic or political, taking the total derivative of (4.1) with respect to ϕ, using $\frac{\partial s_N^*}{\partial \phi} \equiv \frac{\partial n_N^*}{n_W \partial \phi} \equiv -\frac{\partial n_S^*}{n_W \partial \phi}$ and solving for $\frac{\partial s_N^*}{\partial \phi}$ implies that (proof in the Appendix):

Proposition 3: *The establishment of manufacturing production in the economically smaller region is increasing in the level of regional integration, in relation to the location equilibrium without endogenous rent seeking:* $\frac{\partial s_N^*}{\partial \phi} < \frac{\partial s_N}{\partial \phi}$*, whenever* $\frac{\partial s_N}{\partial \phi} > 0$.

According to Proposition 3, liberalizing inter-regional trade thus seems to decrease the relative strength of the agglomeration process when influence activities are taken into account in the analysis of the equilibrium geography: as the two regions become more integrated, regional policy, determined by endogenous rent seeking, becomes progressively more effective and, as a result, increases the number of firms in the politically stronger region, relative to the equilibrium without lobbying. The explanation for this result is that the benefit of producing in the region hosting the larger number of consumers is less crucial for the location decisions of firms when the movement of goods becomes gradually unrestricted and the government allocates a larger share of the rent to the South. For a given distribution of expenditures, subsidies to manufacturing production are therefore relatively more efficient in attracting industrial activity when transaction costs are sufficiently low, since firms then become indifferent to location. This finding is illustrated in Figure 2, which shows how the share of industry in the North changes as trade barriers gradually are lowered, using the following parameter values: $T = 0.3, L_W = 100,000, n_W = 100,000$ (i.e., one unit of labor and capital per variety) and $s_{E_N} = 0.9$. For comparison, the graph also displays (as a dashed curve) the location equilibrium where no net subsidies are transferred ($\theta = 1$), using the same parameter values.

Starting from autarky in Figure 2, during the first stages of trade liberalization (up to ϕ^*) the distribution of industry between the regions stays almost constant, and it follows that the effect of the relative economic strength of the North on the spatial equilibrium then must be nearly offset by the effect of the South's relative political strength. Consequently, freer trade magnifies

Figure 2: The share of industry in the North for different degrees of openness



the establishment of local production in the larger region at a much slower rate compared to the location equilibrium without endogenous rent seeking, thus providing support for Proposition 2. Inter-regional trade protection is, however, sufficiently costly to induce some relocation of firms to the larger region, since $\frac{\partial s_N^*}{\partial \phi} > 0$ for $\phi \in [0, \phi^*)$. In other words, regional outward direct investment, motivated by avoiding trade barriers (tariff-jumping), is still profitable for the representative exporting firm at these transaction costs. Further regional integration results in a shift of manufacturing production towards the region with relative factor scarcity, which, together with the observed change in the location equilibrium where no net subsidies are transferred, provide evidence for Proposition 3; according to Figure 2, the number of firms in the politically stronger region clearly increases as trade becomes freer, in relation to the equilibrium without rent seeking. At this point, where $\frac{\partial s_N^*}{\partial \phi} < 0$ and $\phi \in (\phi^*, 1]$, the relative economic strength of the North is dominated by the South's relative political strength; that is to say, for the manufacturing sector the benefit of a government policy directed towards subsidizing the establishment of industrial firms in the economically smaller region now outweighs the market-access advantage of producing in the larger region.[4] According to Proposition 3, this regional policy becomes more effi-

[4]It can be established that the share of industry in the North implied by the rent-seeking equilibrium is decreasing in the level of economic integration for all $s_{E_N} \in (0.5, 1]$

cient in attracting industrial production as trade barriers are reduced. With free trade, the political economy equilibrium therefore requires that the relocation of manufacturing activities takes place up until all the gains from government subsidies are exhausted, where there are as many firms operating in the South as in the North, and neither of the regions consequently receive any net transfers at the second stage of the political economy game. To show that this is the unique free-trade equilibrium, consider any spatial outcome where $s_N^* > 0.5$, and thus $n_N^* > n_S^*$ such that production in the South is net subsidized by (3.18). Then, any firm could have increased its subsidy-included rate of return by relocating to the South, so this cannot be an equilibrium. The only situation, in which no firm can increase the profit from relocation, is when both regions are symmetric in terms of their capital endowment: $s_N^* = 0.5$, $n_N^* = n_S^*$, and accordingly $\theta_{NE} = 1$ by (3.18). This prediction is indeed confirmed by Figure 2.

Now denote the value of ϕ by ϕ^* that corresponds to the maximum of s_N^* and shown in Figure 2 above, then the condition that determines ϕ^* is (see Appendix A.4 for details):

$$\phi^* = \frac{\theta_{NE}\left(2s_{E_N} - 1\right) + \left(1 - \theta_{NE}^2\right)\sqrt{s_{E_N}\left(1 - s_{E_N}\right)}}{s_{E_N}\left(1 + \theta_{NE}^2\right) - 1}, \tag{4.2}$$

which is defined when $\phi^* \in [0, 1)$. At $\phi = \phi^*$, the relative economic strength of the North is exactly offset by the South's relative political strength, because (4.2) satisfies $\frac{\partial s_N^*}{\partial \phi} = 0$. Moreover, by differentiating (4.2) it can be shown that ϕ^* is decreasing in θ_{NE}, and therefore it takes a less extensive liberalization of trade for capital flows to reverse to the South when the government increases net transfers to the backward region; a redistributive policy that progressively supports manufacturing production in the South implies a lower critical level of transaction costs. It is also straightforward to verify that $\frac{\partial \phi^*}{\partial s_{E_N}} > 0$. Hence, the threshold value increases when the North becomes economically stronger, since this decreases the effectiveness with which regional policy slows down the agglomeration process.

And so, once the rent-seeking dimension is introduced as a determinant of regional policy, the equilibrium location of industry does not seem to depend solely on economic forces. The equilibrium share of manufacturing activities is still increasing in the spatial distribution of expenditures. But, at the same time, economic strength with relative factor abundance and a large

given that $\phi > \phi^*$, and monotonically decreasing for some $s_{E_N} \in (0.5, 1]$.

industry sector will give rise to less net subsidies in equilibrium, which reflects a political weakness; a dollar increment of transfers at the first stage in the political game specified in Section 3 is worth less to agents in the economically larger region, since more of it will be dissipated in the second-stage regional contest. For a given distribution of expenditure shares and with a constant level of inter-regional integration, an increase in net subsidies to firms located in the South decreases the spatial concentration in the North. Consequently, when a model of lobbying for regional benefits is combined with a model of economic geography, the prediction implied by the location equilibrium without rent seeking that the larger region becomes the core when trade barriers are sufficiently low does not hold anymore.[5]

5 Conclusions

This paper has analyzed how the endogenous determination of regional subsidies affects the equilibrium location of manufacturing activities. Once political factors such as rent seeking for government transfers to regions are included in an economic geography framework with size asymmetries, the core-periphery outcome that follows from the economic model, if transaction costs are low enough, no longer holds. Instead, once trade becomes freer than a certain threshold value, the equilibrium location of industry in the smaller region increases in the level of regional integration, whereas the agglomeration process slows down. Intuitively, since a larger share of transfers per firm to the North will be dissipated in the political equilibrium compared to the smaller region, agents in the South are willing to expend relatively more resources for a marginal increment of subsidies to its manufacturing sector. The government therefore distributes a larger share of the rent to the economically smaller region, and this increases the subsidy-included reward to capital in the South. Thus, the South can attract more firms in contrast to the location equilibrium without lobbying for regional transfers, because the home market effect is now partly offset by the fact that in the South there

[5]This finding resembles that of Robert-Nicoud and Sbergami (2004), where the economically smaller region can attract a more than proportional number of firms if it has a sufficiently large number of swing voters. However, and in contrast to the approach taken in this paper, their result is obtained by assuming asymmetric regional ideological preferences: the social and economic activities are by assumption more heterogeneous in the larger region.

is a relatively lower rate of dissipation in the political equilibrium. When free trade prevails, the relocation of manufacturing production takes place up to the point where there are as many firms operating in the South as in the North. Hence, when rent-seeking activities are taken into consideration in the analysis of the equilibrium geography, it is not true that the larger region attracts all the economic activity.

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A Appendix

A.1 Proof of Proposition 1

Equation (4.1) reproduced:

$$s_N^* = f[s_{E_N},\, \phi,\, \theta_{NE}[\Gamma,\, n_N^*(s_{E_N},\, \phi),\, n_S^*(s_{E_N},\, \phi)]]. \tag{A.1}$$

Taking the total derivative of (A.1) with respect to s_{E_N}, using $\frac{\partial s_N^*}{\partial s_{E_N}} \equiv \frac{\partial n_N^*}{n_W \partial s_{E_N}} \equiv -\frac{\partial n_S^*}{n_W \partial s_{E_N}}$, and solving for $\frac{\partial s_N^*}{\partial s_{E_N}}$ to obtain:

$$\frac{\partial s_N^*}{\partial s_{E_N}} = \frac{\frac{\partial f}{\partial s_{E_N}}}{1 + n_W\left(\frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_S^*} - \frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_N^*}\right)} < \frac{\partial f}{\partial s_{E_N}} = \frac{\partial s_N}{\partial s_{E_N}}, \tag{A.2}$$

provided that $\frac{\partial f}{\partial s_{E_N}} > 0$, since $\frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_S^*} > 0$, and $\frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_N^*} < 0$. QED.

A.2 Proof of Proposition 2

Differentiating $\frac{\partial s_N^*}{\partial s_{E_N}}$ with respect to ϕ yields:

$$\begin{aligned}
\frac{\partial^2 s_N^*}{\partial \phi \partial s_{E_N}} &= \frac{\frac{\partial^2 f}{\partial \phi \partial s_{E_N}}}{1 + n_W\left(\frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_S^*} - \frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_N^*}\right)} - \\
&\frac{n_W \frac{\partial f}{\partial s_{E_N}}\left[\frac{\partial^2 f}{\partial \phi \partial \theta_{NE}}\left(\frac{\partial \theta_{NE}}{\partial n_S^*} - \frac{\partial \theta_{NE}}{\partial n_N^*}\right) + \frac{\partial f}{\partial \theta_{NE}}\frac{\partial}{\partial \phi}\left(\frac{\partial \theta_{NE}}{\partial n_S^*} - \frac{\partial \theta_{NE}}{\partial n_N^*}\right)\right]}{\left[1 + n_W\left(\frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_S^*} - \frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_N^*}\right)\right]^2} \\
&< \frac{\partial^2 f}{\partial \phi \partial s_{E_N}} = \frac{\partial^2 s_N}{\partial \phi \partial s_{E_N}},
\end{aligned} \tag{A.3}$$

given that $\frac{\partial^2 f}{\partial \phi \partial s_{E_N}} > 0$, $\frac{\partial f}{\partial s_{E_N}} > 0$, and $\frac{\partial^2 f}{\partial \phi \partial \theta_{NE}} = \frac{\partial^2 f}{\partial \theta_{NE}^2}\left(\frac{\partial \theta_{NE}}{\partial n_N^*}\frac{\partial n_N^*}{\partial \phi} + \frac{\partial \theta_{NE}}{\partial n_S^*}\frac{\partial n_S^*}{\partial \phi}\right) < 0$, because $\frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_S^*} > 0$, $\frac{\partial f}{\partial \theta_{NE}}\frac{\partial \theta_{NE}}{\partial n_N^*} < 0$, $\frac{\partial \theta_{NE}}{\partial n_S^*} < 0$, $\frac{\partial \theta_{NE}}{\partial n_N^*} > 0$, $\frac{\partial f}{\partial \theta_{NE}} < 0$. Now $\frac{\partial}{\partial \phi}\left(\frac{\partial \theta_{NE}}{\partial n_S^*} - \frac{\partial \theta_{NE}}{\partial n_N^*}\right) = \frac{\partial n_N^*}{\partial \phi}\left[2\frac{\partial^2 \theta_{NE}}{\partial n_S^* \partial n_N^*} - \frac{\partial^2 \theta_{NE}}{\partial {n_N^*}^2} - \frac{\partial^2 \theta_{NE}}{\partial {n_S^*}^2}\right] < 0$, as long as $\frac{\partial n_N^*}{\partial \phi} \equiv \frac{n_W \partial s_N^*}{\partial \phi} > 0$, since $2\frac{\partial^2 \theta_{NE}}{\partial n_S^* \partial n_N^*} - \frac{\partial^2 \theta_{NE}}{\partial {n_N^*}^2} - \frac{\partial^2 \theta_{NE}}{\partial {n_S^*}^2} = \frac{6\Gamma[-3{n_S^*}^2 {n_N^*}^2 \Gamma({n_S^*}^3\Gamma+\rho) - n_N({n_S^*}^3\Gamma+\rho)^2 - n_S(2{n_S^*}^3\Gamma-\rho)({n_N^*}^3\Gamma+\rho)]}{({n_S^*}^3\Gamma+\rho)^3} < 0$ if $2{n_S^*}^3\Gamma - \rho > 0$, which is satisfied when $z_N > \frac{1}{2}$, and $z_j \geq 1$. QED.

A.3 Proof of Proposition 3

By totally differentiating (A.1) with respect to ϕ, using $\frac{\partial s_N^*}{\partial \phi} \equiv \frac{\partial n_N^*}{n_W \partial \phi} \equiv -\frac{\partial n_S^*}{n_W \partial \phi}$ and rearranging terms, it follows that:

$$\frac{\partial s_N^*}{\partial \phi} = \frac{\frac{\partial f}{\partial \phi}}{1 + n_W \left(\frac{\partial f}{\partial \theta_{NE}} \frac{\partial \theta_{NE}}{\partial n_S^*} - \frac{\partial f}{\partial \theta_{NE}} \frac{\partial \theta_{NE}}{\partial n_N^*} \right)} < \frac{\partial f}{\partial \phi} = \frac{\partial s_N}{\partial \phi}, \tag{A.4}$$

whenever $\frac{\partial f}{\partial \phi} > 0$, as $\frac{\partial f}{\partial \theta_{NE}} \frac{\partial \theta_{NE}}{\partial n_S^*} > 0$, and $\frac{\partial f}{\partial \theta_{NE}} \frac{\partial \theta_{NE}}{\partial n_N^*} < 0$. QED.

A.4 Determining the Level of Trade Barriers Where Capital Flows Are Reversed

Taking the total derivative of (4.1) with respect to ϕ, using $\frac{\partial s_N^*}{\partial \phi} \equiv \frac{\partial n_N^*}{n_W \partial \phi} \equiv -\frac{\partial n_S^*}{n_W \partial \phi}$ and rearranging terms, gives:

$$\frac{\partial s_N^*}{\partial \phi} = \frac{-(\theta_{NE} - \phi)^2 + s_{E_N}[1 - 4\theta_{NE}\phi + \phi^2 + \theta_{NE}^2(1 + \phi^2)]}{(\phi - 1)^2[\phi - \theta_{NE} + s_{E_N}(\theta_{NE} - 1)(1 + \phi)]^2 \Psi}, \tag{A.5}$$

where $\Psi \equiv \left[1 - \frac{3(s_{E_N} - 1)s_{E_N} n_W \Gamma[n_S^{*2} n_N^{*2}(n_S^* + n_N^*)\Gamma + (n_S^{*2} + n_N^{*2})\rho](1+\phi)^2}{(n_S^{*3}\Gamma + \rho)^2[\phi - \theta_{NE} + s_{E_N}(\theta_{NE} - 1)(1+\phi)]^2}\right]$. $\frac{\partial s_N^*}{\partial \phi} = 0$ if and only if:

$$(\theta_{NE} - \phi)^2 = s_{E_N}[1 - 4\theta_{NE}\phi + \phi^2 + \theta_{NE}^2(1 + \phi^2)], \tag{A.6}$$

for $\phi \in [0, 1)$ and $\phi - \theta_{NE} + s_{E_N}(\theta_{NE} - 1)(1 + \phi) \neq 0$. Now solve (A.6) for the root that yields a global maximum in the admissible parameter space, $s_N^* \in (0, 1)$, to acquire (4.2).